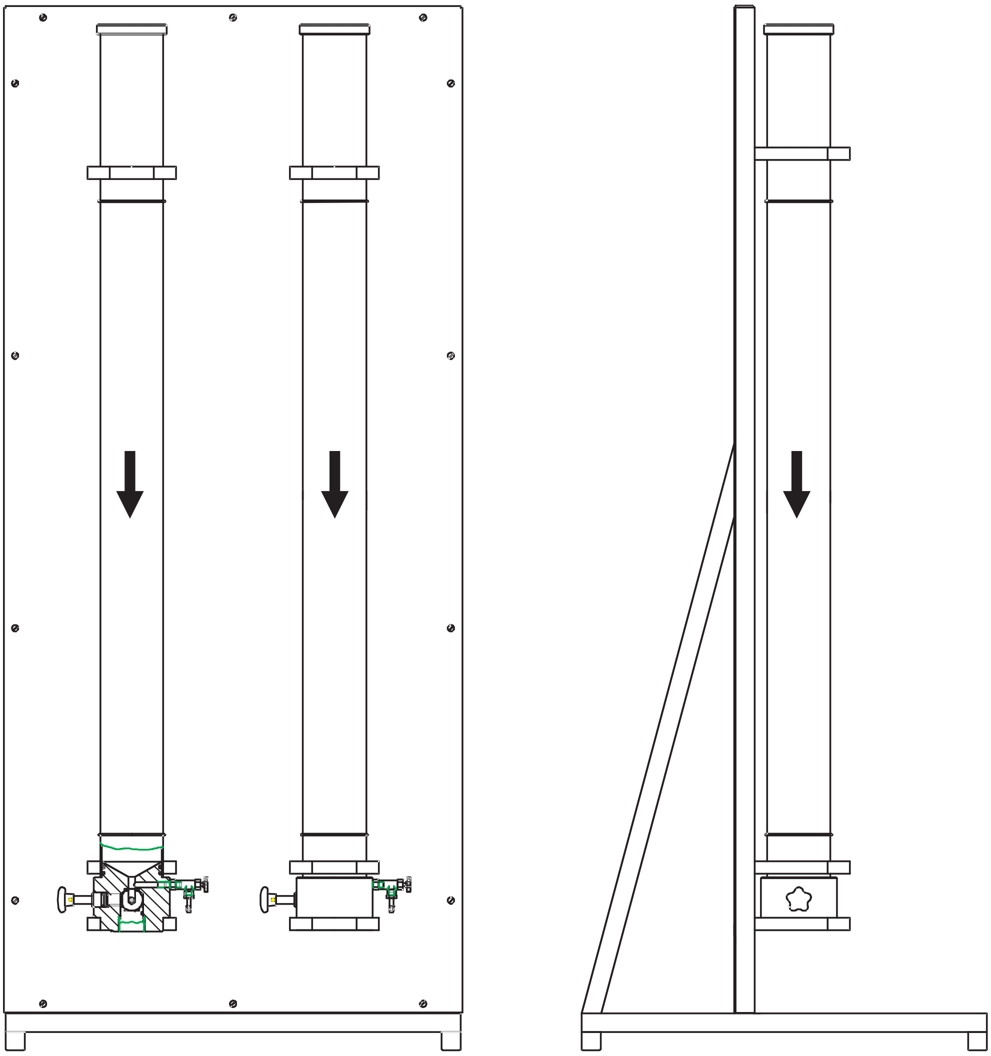
Experiment Instructions

HM 135 Drag Coefficients for Spheres

HM 135 ***DRAG COEFFICIENTS FOR SPHERES***



Experiment Instructions

Please read and follow the instructions before the first installation!

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1 Unit Description

The unit is used to investigate sink speeds of objects in liquids. Spheres made of different materials and volumes are used.

The sphere is guided down a transparent tube filled with liquid. The sphere sinks to the floor under the action of gravity. The sink time is measured for a specific measured section. From this it is possible to derive the mean sink speed. The two measuring tubes can be filled with liquids of different viscosity. In this way it is possible to directly compare the sink speeds

At the bottom of the tube there is a chamber. In this way the sphere can be removed from the measuring tube without large loss of liquid. The chamber has got a device behaving similar to a drawer. It is operated by hand.

Draw out the lever until the limit. Rotate the lever to make the sphere drop out of the slot. Afterwards turn lever to achieve slots upright position again. Push in drawer up to the limit.

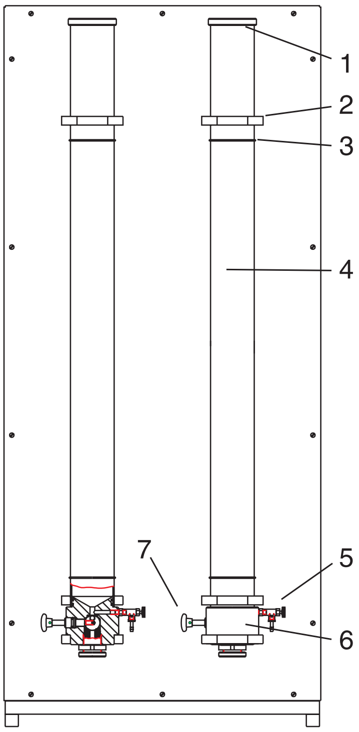

Fig. 1.1

The unit essentially comprises:

Cover with inlet hole (1)

Pipe bracket (2)

2 off o-rings (3)

Measuring tube (4) made of Plexiglass

Drain valve (5)

Chamber (6)

Chamber manual lever (7)

2 Preparations for the Experiments

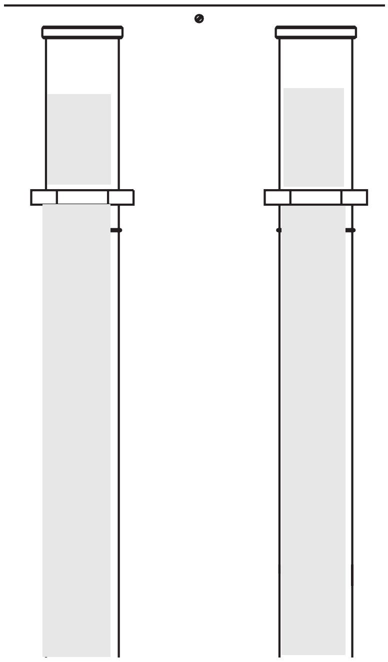

Fig. 2.1

Proceed as follows:

1. Place the experimental setup on a level surface.
2. Fill the two Plexiglass tubes with the liquids Ensure that the drain cocks are closed.
3. Place sphere ready at hand.

The following is also required:

- Stopwatch
- Araeometer
- Letter-scales

3 Experiments

For the comparison with the theory the following data is required:

- Density ρ_F of the liquid
- Density ρ_K and diameter D_K of the related sphere.
- The sink time t of the sphere for the section between the two o-rings used as measuring marks.

The density of the sphere is determined from the volume V_K and the mass m_K.

The distance between the o-rings should be set to h= 1m.

3.1 Performing the Experiment

Various spheres are provided for the experiment.

- Aluminium D = 10 mm, $\rho = 2.86\, g/cm^3$
- Aluminium D = 5 mm, $\rho = 2.86\, g/cm^3$
- POM (Delrin) D = 10 mm, $\rho = 1.43\, g/cm^3$
- POM (Delrin) D= 5 mm, $\rho = 1.43\, g/cm^3$
- PA6.6 (Nylon) D = 10 mm, $\rho = 1.15\, g/cm^3$

The size of the sphere must not exceed a diameter of D_K= 12 ~ 13 mm.
(Risk of blockage of the chamber)

The density should be checked.

$$\rho_K = \frac{m}{V_K}$$

The mass m can be obtained by weighing.
Note: Determine the mass of several spheres. This will increase the accuracy of the measurement.

The force due to weight F is given by $F = m \cdot g$.

The volume of the sphere is calculated as

$$V_K = \frac{\pi \cdot D_K^{\ 3}}{6}$$

The surface area is:

$$A_K = \pi \cdot D_K^{\ 2}$$

The diameter D_K should be measured with a micrometer.

3.2 Measured Values for the Spheres

Measured values for the spheres:

D_K [mm]	Area $[m^2 \cdot 10^{-5}]$	Material	ρ_K [g/cm3]	V_K [cm3]	m [g]	F $[N \cdot 10^{-3}]$
5.0	1.96	POM	1.43	0.065	0.093	0.912
10.0	7.85	POM	1.43	0.523	0.748	7.33
5.0	1.96	Alu	2.86	0.065	0.186	1.82
10.0	7.85	Alu	2.86	0.523	1.50	14.71
10.0	7.85	PE6.6	1.15	0.523	0.6	5.88

3.3 Measured Values for the Sink Speeds

For the medium water (17_C) the following fall times and sink speeds have been determined.

Here the mean was taken from 5 measurements.

The flow speed is calculated

from: $v = \frac{h}{t}$

Material	D_K [mm]	t_1 [s]	t_2 [s]	t_3 [s]	t_4 [s]	t_5 [s]	t_m [s]	v [m/s]
POM	5.0	4.32	4.42	4.42	4.47	4.36	4.40	0.227
POM	10.0	3.27 *	3.24	3.15	3.16	3.26 *	3.22	0.310
Alu	5.0	2.01	1.80	1.81	1.98	1.90	1.90	0.526
Alu	10.0	1.77 *	1.26	1.25	1.47 *	1.24	1.25	0.80
PE6.6	10.0	6.63 *	5.70	5.63	5.85	5.59	5.69	0.175

(* with wall friction)
Wall friction can occur due to tumbling of the sphere. These measured values are not to be taken into account.

4 Theoretical Calculation of the Sink Speed

The following factors are to be taken into account in the calculation of the sink speed.

- Force due to weight of the sphere (downward force) F_{Ab}
- Upward force on the sphere F_{Auf}
- Pressure drag F_{Dw} on the sphere in the liquid (dependent on the cross-sectional area of the sphere)
- Surface friction F_{OR} on the sphere in the liquid (dependent on the surface of the sphere)

The sum of all the forces is equal to zero.

$$\sum F = 0 = F_{Ab} - F_{Auf} - F_{Dw} - F_{Or}$$

with

$$F_{Ab} = \rho_K \cdot V_K \cdot g$$

$$F_{Auf} = \rho_F \cdot V_K \cdot g$$

$$F_{Dw} = \tfrac{1}{2} \cdot \rho_F \cdot cw \cdot v^2 \cdot A_{QK}$$

$$F_{Dr} = \tfrac{1}{2} \cdot \rho_F \cdot cf \cdot v^2 \cdot A_K$$

The c_W value for a sphere is cw=0.22.

The c_f value is calculated as

$$c_f = \frac{1{,}328}{\sqrt{\text{Re}}} \quad \text{with}$$

$$\text{Re} = \frac{D_K \cdot v}{\nu}$$

v = sink speed m/s

ν = kinematic viscosity of the medium

(water at 17 °C, $\nu = 1{,}007 \cdot 10^{-6} \frac{m^2}{s}$

The cross-sectional area of the sphere is

$$A_{QK} = \frac{\pi \cdot D_K{}^2}{4}$$

By inserting the individual relationships the following equation of equilibrium is found

$$V_K \cdot (\rho_K - \rho_F) \cdot g = \tfrac{1}{2} \cdot \rho_F \cdot c_w \cdot v^2 \cdot A_{QK} - \tfrac{1}{2} \cdot \rho_F \cdot c_f \cdot v^2 \cdot A_K$$

This formula must now be solved for v.

This then yields a 4th order polynomial.

$$\frac{D_K{}^2 \cdot g}{9} \cdot \left(\frac{\rho_K}{\rho_F} - 1\right)^2 \cdot x^4 - \frac{D_K \cdot g \cdot c_w}{6} \cdot \left(\frac{\rho_K}{\rho_F} - 1\right) \cdot x^2 - \frac{1{,}328^2 \cdot \nu}{D_K} \cdot x\,D + \frac{c_w{}^2}{16} = 0$$

with $x = \frac{1}{v}$

This relationship must be solved iteratively, e.g., using the numerical method after Newton.

For simplification the following constants are used.

$$A = \frac{D_K^{\ 2} \cdot g}{9} \cdot \left(\frac{\rho_K}{\rho_F} - 1\right)^2$$

$$B = \frac{D_K \cdot g \cdot c_w}{6} \cdot \left(\frac{\rho_K}{\rho_F} - 1\right)$$

$$C = -\frac{1{,}328^2 \cdot \nu}{D_K}$$

$$D = \frac{c_w^{\ 2}}{16}$$

In the way the polynomial takes on the following form:

$$A \cdot x^4 + B \cdot x^2 + C \cdot x + D = 0$$

4.1 Results of the Calculations

The factors for equation, the solutions after Newton and the sink speed measured are listed in the table.

The density of water was taken as 1.0 g/cm3.

Material	D_K [mm]	ρ_K [g/cm3]	A $[m^3/s^2 \cdot 10^{-3}]$	B $[m^2/s^2 \cdot 10^{-3}]$	C $[s/m \cdot 10^{-3}]$	D	X [s/m]	$v = \frac{1}{x}$ [m/s]	v [m/s] as per
POM	5.0	1.43	0.04943	-0.7734	-0.3552	0.003	4.380	0.228	0.227
POM	10.0	1.43	0.1977	-1.547	-0.1776	0.003	3.085	0.324	0.310
Alu	5.0	2.86	0.9248	-3.345	-0.3552	0.003	2.106	0.475	0.526
Alu	10.0	2.86	3.699	-6.669	-0.1776	0.003	1.484	0.674	0.80
PE6.6	10.0	1.15	0.0241	-0.5396	-0.1776	0.003	5.22	0.192	0.175

The measured values for the sink speed vin accordance with match well with the calculated values of v.

A large source of error lies in the determination of the time *t* because this is measured by hand with a stopwatch. This is particularly the case for spheres on higher density (Al). As on these spheres the sink speed is high, human reaction time has a large effect when using the stopwatch.

The measured values for aluminium therefore vary the most from the theoretical values.

5 Technical Data

Dimensions and Weights of the Spheres:

Aluminium	D_K = 10 mm, density 2.86 g/cm^3
Aluminium	D_K = 5 mm, density 2.86 g/cm^3
POM (Delrin)	D_K = 10 mm, density 1.43 g/cm^3
POM (Delrin)	D_K= 5 mm, density 1.43 g/cm^3
PE6.6 (Nylon)	D_K = 10 mm, density 1.15 g/cm^3

Max. Perm. Diameter = 10 mm

Fall Height *H* 1 m
can be changed by sliding the O-rings

Tube Sections:

Length	1330	mm
Ø D/d	100 / 92	mm

Capacity: 8 litres

Overall Dimensions:

W x D x H	720 x 640 x 1650	mm
Total Weight	45	kg