Homework 1

A 25-kVA, 440/220-V, 60-Hz transformer has the following parameters:

$$R_1 = 0.16\ \Omega \qquad R_2 = 0.04\ \Omega \qquad R_{c1} = 270\ \Omega$$
$$X_1 = 0.32\ \Omega \qquad X_2 = 0.08\ \Omega \qquad X_{m1} = 100\ \Omega$$

The transformer delivers 20 kW at 0.8 power factor lagging to a load on the low-voltage side with 220 V across the load. Find the primary terminal voltage.

Solution The voltage across the load is taken as reference phasor; thus,

$$\mathbf{V}_2 = 220\angle 0^\circ \text{ V}$$

For a load $P_2 = 20{,}000$ W at 0.8 power factor lagging, the secondary current is computed as follows:

$$\mathbf{I}_2 = \frac{20{,}000}{(220)(0.8)}\angle -\cos^{-1}0.8 = 113.64\angle -36.9^\circ \text{ A}$$

The transformer turns ratio is $a = 440/220 = 2$. Thus, the secondary voltage and current and the winding resistance and reactance are referred to the primary side as follows:

$$a\mathbf{V}_2 = 2(220\angle 0^\circ) = 440\angle 0^\circ \text{ V}$$
$$\mathbf{I}_2/a = (113.64\angle -36.9^\circ)/2 = 56.82\angle -36.9^\circ \text{ A}$$
$$a^2R_2 = (2)^2(0.04) = 0.16\ \Omega$$
$$a^2X_2 = (2)^2(0.08) = 0.32\ \Omega$$

Referring to the phasor diagram of Fig. 4.13, the primary induced voltage is calculated as follows:

$$\begin{aligned}\mathbf{E}_1 &= a\mathbf{V}_2 + (\mathbf{I}_2/a)(a^2R_2 + ja^2X_2)\\ &= 440\angle 0^\circ + (56.82\angle -36.9^\circ)(0.16 + j0.32)\\ &= 458.2 + j9.07 = 458.3\angle 1^\circ \text{ V}\end{aligned}$$

The shunt branch currents are

$$\mathbf{I}_c = \mathbf{E}_1/R_{c1} = (458.2 + j9.07)/270 = 1.7 + j0.03 \text{ A}$$
$$\mathbf{I}_m = \mathbf{E}_1/jX_{m1} = (458.2 + j9.07)/j100 = 0.09 - j4.58 \text{ A}$$
$$\mathbf{I}_e = \mathbf{I}_c + \mathbf{I}_m = 1.79 - j4.55 \text{ A}$$

Thus, the primary current is

$$\mathbf{I}_1 = \mathbf{I}_e + \mathbf{I}_2/a$$
$$= (1.79 - j4.55) + (56.82\angle -36.9°) = 61.04\angle -39.3° \text{ A}$$

Therefore, the primary voltage is found from

$$\mathbf{V}_1 = \mathbf{E}_1 + \mathbf{I}_1(R_1 + jX_1)$$
$$= (458.2 + j9.07) + (61.04\angle -39.3°)(0.16 + j0.32)$$
$$= 478.1 + j18 = 478.4\angle 2.2° \text{ V}$$