

بِسْمِ اللَّهِ الرَّحْمَنِ الرَّحِيمِ

Kingdom of Saudi Arabia
Ministry of Higher Education
Al-Imam Mohammed Ibn Saud
Islamic University
- College of Science -
Department: Maths
Semester/Year: 1431-1432
Duration: 2 hours

المملكة العربية السعودية
وزارة التعليم العالي
جامعة الإمام محمد بن سعود الإسلامية
كلية العلوم -
Course Name: Calculus 2
Course Code: Math 102
Drs. S. Mziou and M. F. Bellalouna

Exam

Question 1. [3+3+3 marks]

Evaluate the following integrals.

1. $\int_0^1 \frac{3x - 1}{x^2 + x - 6} dx,$
2. $\int_0^{+\infty} \frac{x e^{-x^2}}{1 + e^{-x^2}} dx .$

Question 2. [3+3+3 marks]

Investigate the convergence or divergence of the following series.

1. $\sum_{k=0}^{\infty} \frac{k^3}{3k^3 - 2k + 1} ,$
2. $\sum_{k=1}^{\infty} \frac{2^k}{k 3^k} ,$

Question 3. [4 marks]

Find the area between the curves of the two functions f and g , where

$$f(x) = x^2 - x, \quad g(x) = 2x - 2 .$$

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Question 4. [3+4+1 marks]

1. Calculate the exact value of the integral $\int_1^5 x \ln(x) dx$.
2. Use the trapezoidal method, with $n = 4$, to obtain an approximation of this integral.
3. Give the absolute error made by using this approximation.

Question 5. [3+3+2+2 marks]

1. Find the radius of convergence and the interval on which the following power series is convergent.

$$\sum_{k=0}^{\infty} 2^k x^k.$$

2. Show that the sum of the following (geometric) series is given by:

$$\sum_{k=0}^{\infty} 2^k x^k = \frac{1}{1-2x}, \quad |x| < \frac{1}{2}.$$

3. Deduce the sums of the following power series.

$$(a) \quad \sum_{k=1}^{\infty} 2^k k x^{k-1}, \quad (b) \quad \sum_{k=0}^{\infty} \frac{2^k}{k+1} (x+1)^k.$$