May 9, 2011 Semester 2, 1431-1432 (2010-20111)

MATH 251 - Introduction to MATLAB

Exercise sheet 6

Dr. Samy MZIOU

Exercise 1 :

1. Write a function file **R = RECTANGLE(a, b)** which compute the area of a rectangle with length a and width b.
2. Write a function file which compute the area of a trapezoid.
3. Write a function that compute the maximum of a vector $VECT$

Exercise 2 :

6.10 Examples of MATLAB Applications

Sample Problem 6-4: Exponential growth and decay

A model for exponential growth, or decay, of a quantity is given by:

$$A(t) = A_0 e^{kt}$$

where $A(t)$ and A_0 are the quantity at time t and time 0, respectively, and k is a constant unique to the specific application.

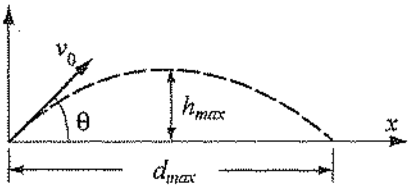
Write a user-defined function that uses this model to predict the quantity $A(t)$ at time t from knowing A_0 and $A(t_1)$ at some other time t_1. For function name and arguments use `At = expGD(A0,At1,t1,t)`, where the output argument `At` corresponds to $A(t)$, and the input arguments `A0,At1,t1,t` corresponds to A_0, $A(t_1)$, t_1, and t, respectively.

a) The population of Mexico was 67 millions in the year 1980 and 79 million in 1986. Estimate the population in 2000.

b) The half-life of a radioactive material is 5.8 years. How much of a 7-gram sample will be left after 30 years.

Sample Problem 6-5: Motion of a Projectile

Create a function file that calculates the trajectory of a projectile. The inputs to the function are the initial velocity and the angle at which the projectile is fired. The outputs from the function are the maximum height and distance. In addition, the function generates a plot of the trajectory. Use the function to calculate the trajectory of a projectile that is fired at a velocity of 230 m/s at an angle of 39°.



Solution

The motion of a projectile can be analyzed by considering the horizontal and vertical components. The initial velocity v_0 can be resolved into horizontal and vertical components:

$$v_{0x} = v_0\cos(\theta) \quad \text{and} \quad v_{0y} = v_0\sin(\theta)$$

In the vertical direction the velocity and position of the projectile are given by:

$$v_y = v_{0y} - gt \quad \text{and} \quad y = v_{0y}t - \frac{1}{2}gt^2$$

The time it takes the projectile to reach the highest point ($v_y = 0$) and the corresponding height are given by:

$$t_{hmax} = \frac{v_{0y}}{g} \quad \text{and} \quad h_{max} = \frac{v_{0y}^2}{2g}$$

The total flying time is twice the time it takes the projectile to reach the highest point, $t_{tot} = 2t_{hmax}$. In the horizontal direction the velocity is constant, and the position of the projectile is given by:

$$x = v_{0x}t$$

In MATLAB notation the function name and arguments are taken as: `[hmax,dmax] = trajectory(v0,theta)`. The function file is:

Exercise 3 : — Problem 7-2: Calculating worker's pay —
A worker is paid according to his hourly wage up to 40 hours, and 50 % more for overtime. Write a program in a script file that calculates the pay to a worker. The program asks the user to enter the number of hours and the hourly wage. The program then displays the pay.

Exercise 4 : — Problem 7-4: Converting units of energy —

Write a program in a script file that converts a quantity of energy (work) given in units of either Joule, ft-lb, cal, or eV to the equivalent quantity in different units specified by the user. The program asks the user to enter the quantity of energy, its current units, and the new desired units. The output is the quantity of energy in the new units. The conversion factors are: 1 J = 0.738 ft-lb = 0.239 cal = 6.24×10^{18} eV. Use the program to:

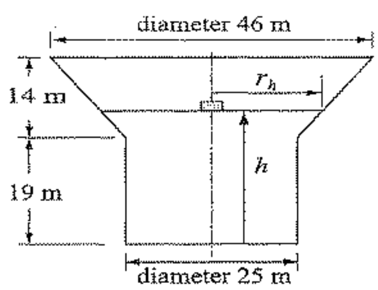
a) Convert 150 J to ft-lb.

b) 2800 cal to Joules.

c) 2.7 eV to cal.

Sample Problem 7-3: Water level in water tower

The tank in a water tower has the geometry shown in the figure (the lower part is a cylinder and the upper part is an inverted frustum cone). Inside the tank there is a float that indicates the level of the water. Write a user-defined function file that determines the volume of the water in the tank from the position (height h) of the float. The input to the function is the value of h in m, and the output is the volume of the water in m³.



Sample Problem 7-5: Sum of series

a) Use a `for-end` loop in a script file to calculate the sum of the first n terms of the series: $\sum_{k=1}^{n} \frac{(-1)^k k}{2^k}$. Execute the script file for $n = 4$ and $n = 20$.

b) The function sin(x) can be written as a Taylor series by:

$$\sin x = \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k+1}}{(2k+1)!}$$

Write a user-defined function file that calculates sin(x) by using the Taylor's series. For the function name and arguments use `y = Tsin(x,n)`. The input arguments are the angle `x` in degrees, and `n` the number of terms in the series. Use the function to calculate sin(150°) using 3 and 7 terms.

Sample Problem 7-6: Modify vector elements

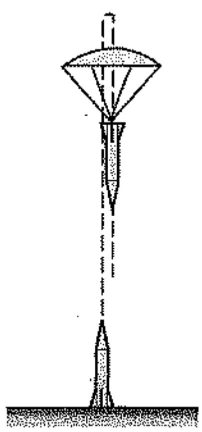
A vector is given by: V = [5, 17, –3, 8, 0, –1, 12, 15, 20, –6, 6, 4, –7, 16]. Write a program as a script file that doubles the elements that are positive and are divisible by 3 and/or 5, and raise to the power of 3 the elements that are negative but greater than –5.

Sample Problem 7-8: Creating a matrix with a loop

Write a program in a script file that creates a $n \times m$ matrix with elements that have the following values. The value of the elements in the first row is the number of the column. The value of the element in the first column is the number of the row. The rest of the elements are equal to the sum of the element above them and the element to the left. When executed, the program asks the user to enter values for n and m.

Sample Problem 7-11: Flight of a model rocket

The flight of a model rocket can be modeled as follows. During the first 0.15s the rocket is propelled up by the rocket engine with a force of 16N. The rocket then flies up while slowing down under the force of gravity. After it reaches the apex, the rocket starts to fall back down. When its down velocity reaches 20 m/s a parachute opens (assumed to open instantly) and the rocket continues to move down at a constant speed of 20 m/s until it hits the ground. Write a program that calculates and plots the speed and altitude of the rocket as a function of time during the flight.



Solution

Exercise 5 : — linear algebra—

let $A = [2, 5, 1; 0, 3, -1]$; $B = [1, 0, 2; -1, 4, -2; 5, 2, 1]$

1. Compute AB
2. Compute BA. explain what happens.
3. Compute the transpose of A

Exercise 6 : — linear algebra—

Consider the following linear system

$$\begin{array}{rcrcrcr} 3x & + & 2y & - & z & = & 10 \\ -x & + & 3y & + & 2z & = & 5 \\ x & - & y & - & z & = & -1 \end{array}$$

1. Write the system using matrix notation
2. Solve the system using **inv** MATLAB command
3. Solve the system using the MATLAB operator \

Exercise 7 : — Interpolation—

Let the following table of points

1	2	3	4	5	6
0	20	55	65	100	120

Find the interpolating polynomial using the MATLAB function **interp1**

Exercise 8 : — Polynomials evaluation—

let the polynomial $p(x) = x^5 - 12.1x^4 + 40.59x^3 - 17.01x^2 - 71.95x + 35.88$,

1. Calculate p(9) using the MATLAB function **polyval**
2. Plot the polynomial for $-1.5 \leq x \leq 6.7$
3. Compute the roots of p using the MATLAB function **roots**

Exercise 9 : — Integration and differentiation—

1. Using MATLAB function **quad** compute the integral of $f(x) = \frac{1}{1+x^2}$ over $[0, 1]$. Compare with the analytical solution.
2. Compute the integral of $f(x) = \frac{1}{1+x^5}$ over $[0, 2]$.

KINGDOM OF SAUDI ARABIA
Ministry of Higher Education
Al-Imam Muhammad Ibn Saud Islamic University
College of Science – Department of Mathematics

المملكة العربية السعودية
وزارة التعليم العالي
جامعة الإمام محمد بن سعود الإسلامية
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FINAL EXAM - Solutions

SEMESTER: SECOND

YEAR: 1430/1431 (2009/2010)

COURSE: STA 111

LEVEL: 3rd & 4th

SECTIONS: 174, 175, 176, 177, 191

DATE: 09/07/1431 (21/06/2010)

DURATION: 2 HOURS

Instructors: Drs. H. FAIRES & F. BELLALOUNA & A. BERKAOUI & A. S. BEN GHORBAL

Name ______________________ Section ______

Student ID ______________ Your Signature ______________

	TOTAL MARKS	SCORE
EXERCISE 1	09.00	
EXERCISE 2	15.00	
EXERCISE 3	15.00	
EXERCISE 4	03.00	
TOTAL	42.00	

Solutions

Sample

INSTRUCTIONS

- Please check that your exam contain **13 Pages** total (including the first page!! & Normal Tables) and **04 EXERC1SES**.
- Please check your test for completeness.
- Read instructions for each problem carefully.
- NO book, NO notes but you may use a calculator which does not graph and which is not programmable.
- Show all your work to get full credit. You must explain how you get your answers using techniques developed in this class so far. Answer with no supporting work, obtained by guess-and-check, or via other methods will result in little or no credit, even correct.
- **Place a box** around **Your Final Answer** to each question.
- Answer the equation in the space provided on the question shcets. If you need more room (space), use the backs of the pages and indicate to the reader that you have done so.
- If you are not sure what a question means, raise your hand and ask me.
- Check your work!

Good Luck!

مع دعائنا لكم بالتوفيق

EXERCISE 1. (09.00 Marks)

PART A (04.50 Marks) A multi-choice exam is composed of **10 questions**; each question consists of **4 possible answers** such that only one is true. The student chooses randomly an answer for each question. Find the probability that:

1. (01.50 Marks) He has exactly 5 correct answers.

Let X be the random variable denoting the number of the Correct answers. Thus $X \sim Bin(n = 10, k, p = \frac{1}{4})$

X is Binomial distribution where

$$P(X = k) = \binom{10}{k} \left(\frac{1}{4}\right)^k \times \left(1 - \frac{1}{4}\right)^{10-k}$$

So $P(X = 5) = \binom{10}{5} \times \left(\frac{1}{4}\right)^5 \times \left(\frac{3}{4}\right)^5 = 0.058$

2. (01.50 Marks) He has at most 9 correct answers.

We have

$$P(X \leq 9) = 1 - P(X = 10)$$
$$= 1 - \left(\frac{1}{4}\right)^{10}$$
$$\simeq 0.999\ldots = 1$$

3. (01.50 Marks) He has at least 2 correct answers.

We have

$$P(X \geq 2) = 1 - P(X < 2)$$
$$= 1 - P(X \leq 1)$$
$$= 1 - P(X = 0) - P(X = 1)$$
$$= 1 - \binom{10}{0}\left(\frac{1}{4}\right)^0 \times \left(\frac{3}{4}\right)^{10-0} - \binom{10}{1}\left(\frac{1}{4}\right) \times \left(\frac{3}{4}\right)^9$$
$$= 0.756$$

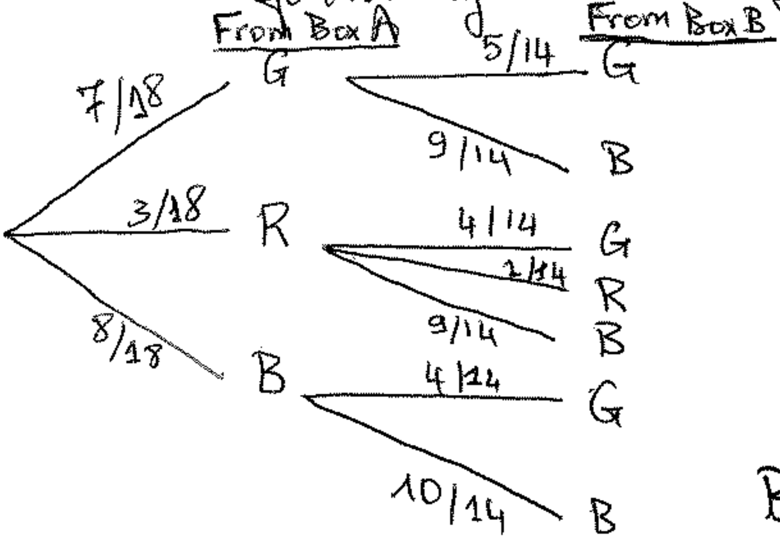
PART B (04.50 Marks) Two boxes A and B are as follows:

- Box A contains 7 green balls, 3 red balls and 8 black balls,
- Box B contains 4 green balls and 9 black balls,

One ball is taken randomly from box *A* (ball 1) and put in box B and then one ball is taken randomly from box *B* (ball 2).

(a) (00.75 Marks) Find the probability that the ball 1 is black.

We have the following tree diagram



B: Black Ball

R: Red Ball

G: Green Ball

Denote:

B_1: Ball 1 is black

B_2: Ball 2 is Black

G_1: Ball 1 is Green

R_1: Ball 1 is Red

Thus $P(\text{Ball 1 is black}) = \dfrac{8}{18}$

(b) (01.50 Marks) Find the probability that the ball 2 is black.

We have

$$P(\text{Ball 2 is black}) = P(B_2|B_1)P(B_1) + P(B_2|G_1)P(G_1) + P(B_2|R_1)P(R_1)$$

$$= \frac{10}{14}\times\frac{8}{18} + \frac{9}{14}\times\frac{7}{18} + \frac{9}{14}\times\frac{3}{18} = \frac{80+63+27}{14\times 18}$$

$$\simeq 0.675$$

(c) (02.25 Marks) Find the probability that the ball 1 is green, given that the ball 2 is black.

We have

$$P(G_1|B_2) = \frac{P(B_2|G_1)P(G_1)}{P(B_2)}$$

$$= \frac{\frac{9}{14}\times 7/18}{\frac{170}{14\times 18}} = \frac{9\times 7}{170} \simeq 0.371$$

EXERCISE 2. (15.00 Marks)

PART A (04.50 Marks) The waiting time, in hours, between successive speeders spotted by a radar unit is a continuous random variable X with cumulative distribution function

$$F_X(x) = \begin{cases} 1-e^{-8x} & \text{if } x \geq 0 \\ 0 & \text{if } x < 0 \end{cases}$$

1. (02.00 Marks) Find the density distribution function f_X of the random variable X.

Since F_X is continuous function by intervals then X is a continuous random variable with probability density function F'_X where

$$F'_X(x) = f_X(x) = \begin{cases} 8e^{-8x} & \text{if } x > 0 \\ 0 & \text{if } x \leq 0 \end{cases}$$

2. (01.00 Marks) Find the probability that the waiting time is lesser than 2 hours, $P(X \leq 2)$.

We have

$$P(X \leq 2) = F_X(2) = 1 - e^{-16} \simeq 1$$

3. (01.50 Marks) Find the probability that the waiting time is bigger than 1 hour, $P(X \geq 1)$.

We have

$$\begin{aligned} P(X \geq 1) &= 1 - P(X < 1) \\ &= 1 - F_X(1) \\ &= 1 - (1 - e^{-8}) = e^{-8} \end{aligned}$$

PART B (03.00 Marks) A continuous random variable X has the following density distribution function

$$f_X(x) = \begin{cases} kx & 0 \le x \le 1 \\ 0 & elsewhere \end{cases}$$

Find the followings:

1. (02.00 Marks) The value of the parameter k.

Since f_X is the p.d.f. of the r.v. X then

$$\int_{-\infty}^{+\infty} f_X(x) = 1 \iff \int_0^1 kx\,dx = 1$$

$$\iff k\left[\frac{x^2}{2}\right]_0^1 = 1$$

$$\iff \frac{k}{2} = 1 \iff k = 2$$

2. (03.00 Marks) The cumulative distribution function F_X, of the random variable X.

We have $f_X(x) = \begin{cases} 2x & \text{if } 0 \le x \le 1 \\ 0 & \text{elsewhere} \end{cases}$

By the definition of c.d.f. we have

$$F_X(x) = \int_{-\infty}^{x} f_X(t)\,dt$$

Then

$$F_X(x) = \begin{cases} 0 & \text{if } x \le 0 \\ x^2 & \text{if } 0 \le x < 1 \\ 1 & \text{if } x \ge 1 \end{cases}$$

3. (01.00 Marks) $P(X \le 0.5)$,

We have $P(X \le 0.5) = F_X(0.5)$
$= (0.5)^2$
$= 0.25$

4. (01.50 Marks) $P(-0.4 \le X \le 0.4)$,

We have $P(-0.4 \le X \le 0.4) = P(X \le 0.4) - P(X \le -0.4)$
$= F_X(0.4) - F_X(-0.4)$
$= (0.4)^2 - 0$
$= 0.16$

5. (03.00 Marks) The mean and the variance of X.

X is a Continuous r.v.

We have : the mean

$$E[X] = \int_{-\infty}^{+\infty} x f_X(x)\,dx = \int_0^1 2x^2\,dx = \frac{2}{3}$$

We have : the variance

$$V[X] = E[X^2] - (E[X])^2$$
$$= \int_{-\infty}^{+\infty} x^2 f_X(x)\,dx - \left(\frac{2}{3}\right)^2$$
$$= \int_0^1 2x^3\,dx - \frac{4}{9}$$
$$= \frac{1}{2} - \frac{4}{9} = \frac{9-8}{18} = \frac{1}{18}$$

EXERCISE 3. (15.00 Marks)

PART A (04.50 Marks) Let Z be a standard normal random variable and calculate the following probabilities:

(a) (00.50 Mark) $P(Z \le 1.32)$

We have $P(Z \le 1.32) = 0.9066$

(b) (01.00 Mark) $P(Z \ge -1.70)$

$$P(Z \geq -1.70) = 1 - P(Z \leq -1.70)$$
$$= 1 - 0.0446$$
$$= 0.9554$$

(c) (01.50 Marks) $P(1.50 \le Z \le 2.50)$

We have $P(1.50 \leq Z \leq 2.50) = P(Z \leq 2.50) - P(Z \leq 1.50)$
$$= 0.9938 - 0.9332$$
$$= 0.0606$$

(d) (01.50 Marks) $P(|Z| \le 2.40)$

We have

$$P(|Z| \leq 2.40) = P(-2.40 \leq Z \leq 2.40)$$
$$= P(Z \leq 2.40) - P(Z \leq -2.40)$$
$$= 0.9918 - 0.0082$$
$$= 0.9836$$

PART B (06.00 Marks) A continuous random variable X has the normal distribution with mean $\mu = 1$ and standard deviation $\sigma = 2$. Compute the following probabilities by standardizing

a) (01.00 Marks) $P(X \le 3.5)$,

Since $X \sim \mathcal{N}(\mu = 1, \sigma^2 = 4)$ then we consider the standarized Normal Distribution $Z = \frac{X-\mu}{\sigma} = \frac{X-1}{2} \sim \mathcal{N}(1,0)$

$$P(X \leq 3.5) = P\left(Z \leq \frac{3.5-1}{2}\right) = P(Z \leq 1.25) = 0.8944$$

b) (01.50 Marks) $P(-2 \le X \le 4.4)$

$$P(-2 \leq X \leq 4.4) = P\left(\frac{-2-1}{2} \leq Z \leq \frac{4.4-1}{2}\right) = P\left(-\frac{3}{2} \leq Z \leq \frac{3.4}{2}\right)$$

$$= P(Z \leq 1.70) - P(Z \leq -0.50)$$

$$= 0.9554 - 0.3085 = 0.6469$$

c) (01.50 Marks) $P(X \ge 4.24)$

$$P(X \geq 4.24) = 1 - P(X \leq 4.24)$$

$$= 1 - P\left(Z \leq \frac{4.24-1}{2}\right)$$

$$= 1 - P(Z \leq 1.62)$$

$$= 1 - 0.9474 = 0.0526$$

d) (02.00 Marks) $P(|X - 8.75| \le 10)$

$$P(|X-8.75| \leq 10) = P(-10 \leq X - 8.75 \leq 10)$$

$$= P(-1.25 \leq X \leq 18.75)$$

$$= P\left(\frac{-1.25-1}{2} \leq Z \leq \frac{18.75-1}{2}\right)$$

$$= P(-1.125 \leq Z \leq 8.875)$$

$$= P(-1.13 \leq Z \leq 8.88)$$

$$= P(Z \leq 8.88) - P(Z \leq -1.13)$$

$$\simeq 1 - 0.1292$$

$$\simeq 0.8708$$

PART C Suppose only 90% of all drivers in Florida regularly wear a seatbelt. A random sample of 100 drivers is selected. Let X be the random variable denoting the number of drivers wearing a seatbelt.

1. (02.00 Marks) What is the probability distribution of the random variable X?

Since any driver will wear a seat belt or not then X is a Binomial Distribution where

$X \sim Bin(100, 90\%)$ with $n = 100$ sample size

$p = 0.90$ probability

2. Approximate the probabilities that

(a) (01.50 Marks) Between 89 and 95 of the drivers in the sample regularly wear a seatbelt?

Since $X \sim Bin(100, 0.90)$ with sample size $n = 100 > 30$

then $X \sim \mathcal{N}(\mu_X, \sigma_X^2)$ where $\mu_X = E[X] = np = 90$

The Normal approximation to the Binomial $\sigma_X^2 = npq = 100 \times 0.90 \times 0.1$

$= 9$

So

$P(83 \leq X \leq 95) = P\left(\frac{83-90}{3} \leq Z \leq \frac{95-90}{3}\right)$ $\sigma_X = 3$

$Z = \frac{X-90}{3} \sim \mathcal{N}(0,1)$

$= P(-2.33 \leq Z \leq 1.66)$

$= P(Z \leq 1.66) - P(Z \leq -2.33) = 0.9515 - 0.0099$

$= 0.9416$

(b) (01.00 Marks) Fewer than 93 of those in the sample regularly wear a seatbelt?

$P(X < 93) = P\left(Z \leq \frac{93-90}{3}\right)$

$= P(Z \leq 1.00)$

$= 0.8413$

EXERCISE 4. (03.00 Marks)

Assume that each of your calls to a popular radio station has a probability of 0.02 of connecting, that is, of not obtaining a busy signal. Assume that your calls are independent.

a) (01.50 Marks) What is the probability that your first call that connects is your **tenth call**?

Let X be the random variable denoting the number of calls until the first connection.

So $X \sim Geom(0.02)$: Geometric distribution with parameter $0 < p < 1$

Thus $P(X = k) = (1-p)^{k-1} p$

and

$$P(X = 10) = (1-0.02)^9 \times 0.02 = (0.98)^9 \times 0.02$$

$$= 0.0167$$

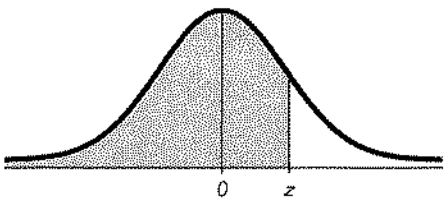
b) (01.50 Marks) What is the probability that it requires more than **three calls** for you to connect?

$$P(X > 3) = 1 - P(X \leq 2)$$

$$= 1 - (P(X = 1) + P(X = 2))$$

$$= 1 - 0.02 - 0.98 \times 0.02$$

$$= 0.9604$$



POSITIVE z Scores

TABLE A-2 (continued) Cumulative Area from the LEFT

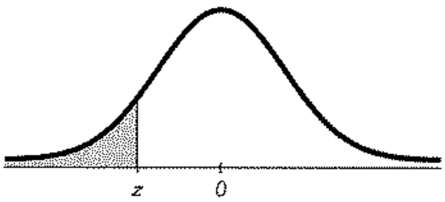
z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495 *	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949 *	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998
3.50 and up	.9999									

NOTE: For values of z above 3.49, use 0.9999 for the area.
*Use these common values that result from interpolation:

z score	Area
1.645	0.9500
2.575	0.9950

Common Critical Values

Confidence Level	Critical Value
0.90	1.645
0.95	1.96
0.99	2.575



NEGATIVE z Scores

TABLE A-2 Standard Normal (z) Distribution: Cumulative Area from the LEFT

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.50 and lower	.0001									
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051 *	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505 *	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
−0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
−0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

NOTE: For values of z below −3.49, use 0.0001 for the area.
*Use these common values that result from interpolation:

z score	Area
−1.645	0.0500
−2.575	0.0050

May 9, 2011 Semester 2, 1431-1432 (2010-20111)

MATH 251 - Introduction to MATLAB

Exercise sheet 6

Dr. Samy MZIOU

Exercise 1 :

1. Write a function file $\mathbf{R} = \mathbf{RECTANGLE}(\mathbf{a}, \mathbf{b})$ which compute the area of a rectangle with length a and width b.
2. Write a function file which compute the area of a trapezoid.
3. Write a function that compute the maximum of a vector $VECT$

Exercise 2 :

6.10 EXAMPLES OF MATLAB APPLICATIONS

Sample Problem 6-4: Exponential growth and decay

A model for exponential growth, or decay, of a quantity is given by:

$$A(t) = A_0 e^{kt}$$

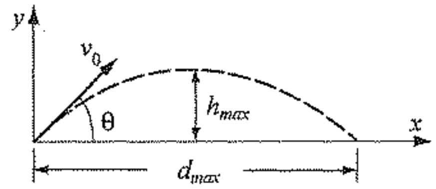
where $A(t)$ and A_0 are the quantity at time t and time 0, respectively, and k is a constant unique to the specific application.

Write a user-defined function that uses this model to predict the quantity $A(t)$ at time t from knowing A_0 and $A(t_1)$ at some other time t_1. For function name and arguments use `At = expGD(A0,At1,t1,t)`, where the output argument `At` corresponds to $A(t)$, and the input arguments `A0,At1,t1,t` corresponds to A_0, $A(t_1)$, t_1, and t, respectively.

a) The population of Mexico was 67 millions in the year 1980 and 79 million in 1986. Estimate the population in 2000.

b) The half-life of a radioactive material is 5.8 years. How much of a 7-gram sample will be left after 30 years.

Sample Problem 6-5: Motion of a Projectile



Create a function file that calculates the trajectory of a projectile. The inputs to the function are the initial velocity and the angle at which the projectile is fired. The outputs from the function are the maximum height and distance. In addition, the function generates a plot of the trajectory. Use the function to calculate the trajectory of a projectile that is fired at a velocity of 230 m/s at an angle of 39°.

Solution

The motion of a projectile can be analyzed by considering the horizontal and vertical components. The initial velocity v_0 can be resolved into horizontal and vertical components:

$$v_{0x} = v_0 \cos(\theta) \quad \text{and} \quad v_{0y} = v_0 \sin(\theta)$$

In the vertical direction the velocity and position of the projectile are given by:

$$v_y = v_{0y} - gt \text{ and } y = v_{0y}t - \frac{1}{2}gt^2$$

The time it takes the projectile to reach the highest point ($v_y = 0$) and the corresponding height are given by:

$$t_{hmax} = \frac{v_{0y}}{g} \text{ and } h_{max} = \frac{v_{0y}^2}{2g}$$

The total flying time is twice the time it takes the projectile to reach the highest point, $t_{tot} = 2t_{hmax}$. In the horizontal direction the velocity is constant, and the position of the projectile is given by:

$$x = v_{0x}t$$

In MATLAB notation the function name and arguments are taken as: `[hmax,dmax] = trajectory(v0,theta)`. The function file is:

Exercise 3 : — Problem 7-2: Calculating worker's pay —
A worker is paid according to his hourly wage up to 40 hours, and 50 % more for overtime. Write a program in a script file that calculates the pay to a worker. The program asks the user to enter the number of hours and the hourly wage. The program then displays the pay.

Exercise 4 : — Problem 7-4: Converting units of energy —

Write a program in a script file that converts a quantity of energy (work) given in units of either Joule, ft-lb, cal, or eV to the equivalent quantity in different units specified by the user. The program asks the user to enter the quantity of energy, its current units, and the new desired units. The output is the quantity of energy in the new units. The conversion factors are: 1 J = 0.738 ft-lb = 0.239 cal = 6.24×10^{18} eV. Use the program to:

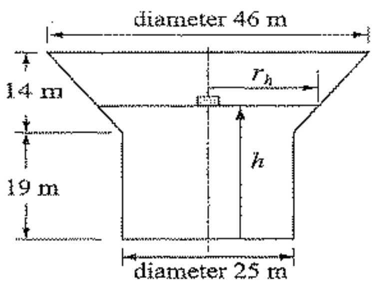
a) Convert 150 J to ft-lb.

b) 2800 cal to Joules.

c) 2.7 eV to cal.

Sample Problem 7-3: Water level in water tower

The tank in a water tower has the geometry shown in the figure (the lower part is a cylinder and the upper part is an inverted frustum cone). Inside the tank there is a float that indicates the level of the water. Write a user-defined function file that determines the volume of the water in the tank from the position (height h) of the float. The input to the function is the value of h in m, and the output is the volume of the water in m^3.



Sample Problem 7-5: Sum of series

a) Use a `for-end` loop in a script file to calculate the sum of the first n terms of the series: $\sum_{k=1}^{n} \frac{(-1)^k k}{2^k}$. Execute the script file for $n = 4$ and $n = 20$.

b) The function sin(x) can be written as a Taylor series by:

$$\sin x = \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k+1}}{(2k+1)!}$$

Write a user-defined function file that calculates sin(x) by using the Taylor's series. For the function name and arguments use `y = Tsin(x,n)`. The input arguments are the angle `x` in degrees, and `n` the number of terms in the series. Use the function to calculate sin(150°) using 3 and 7 terms.

Sample Problem 7-6: Modify vector elements

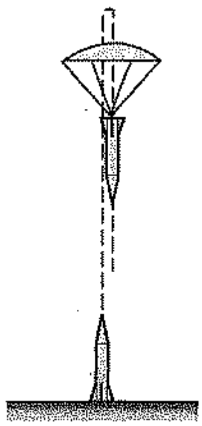
A vector is given by: V = [5, 17, –3, 8, 0, –1, 12, 15, 20, –6, 6, 4, –7, 16]. Write a program as a script file that doubles the elements that are positive and are divisible by 3 and/or 5, and raise to the power of 3 the elements that are negative but greater than –5.

Sample Problem 7-8: Creating a matrix with a loop

Write a program in a script file that creates a $n \times m$ matrix with elements that have the following values. The value of the elements in the first row is the number of the column. The value of the element in the first column is the number of the row. The rest of the elements are equal to the sum of the element above them and the element to the left. When executed, the program asks the user to enter values for n and m.

Sample Problem 7-11: Flight of a model rocket

The flight of a model rocket can be modeled as follows. During the first 0.15s the rocket is propelled up by the rocket engine with a force of 16N. The rocket then flies up while slowing down under the force of gravity. After it reaches the apex, the rocket starts to fall back down. When its down velocity reaches 20 m/s a parachute opens (assumed to open instantly) and the rocket continues to move down at a constant speed of 20 m/s until it hits the ground. Write a program that calculates and plots the speed and altitude of the rocket as a function of time during the flight.



Solution

Exercise 5 : — linear algebra—

let $A = [2, 5, 1; 0, 3, -1]$; $B = [1, 0, 2; -1, 4, -2; 5, 2, 1]$

1. Compute AB
2. Compute BA. explain what happens.
3. Compute the transpose of A

Exercise 6 : — linear algebra—

Consider the following linear system

$$\begin{array}{rcrcrcr} 3x & + & 2y & - & z & = & 10 \\ -x & + & 3y & + & 2z & = & 5 \\ x & - & y & - & z & = & -1 \end{array}$$

1. Write the system using matrix notation
2. Solve the system using **inv** MATLAB command
3. Solve the system using the MATLAB operator \

Exercise 7 : — Interpolation—

Let the following table of points

1	2	3	4	5	6
0	20	55	65	100	120

Find the interpolating polynomial using the MATLAB function **interp1**

Exercise 8 : — Polynomials evaluation—

let the polynomial $p(x) = x^5 - 12.1x^4 + 40.59x^3 - 17.01x^2 - 71.95x + 35.88$,

1. Calculate p(9) using the MATLAB function **polyval**
2. Plot the polynomial for $-1.5 \leq x \leq 6.7$
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Exercise 9 : — Integration and differentiation—

1. Using MATLAB function **quad** compute the integral of $f(x) = \frac{1}{1+x^2}$ over $[0, 1]$. Compare with the analytical solution.
2. Compute the integral of $f(x) = \frac{1}{1+x^5}$ over $[0, 2]$.

May 9, 2011 Semester 2, 1431-1432 (2010-20111)

MATH 251 - Introduction to MATLAB

Exercise sheet 6

Dr. Samy MZIOU

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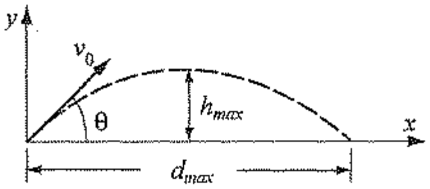

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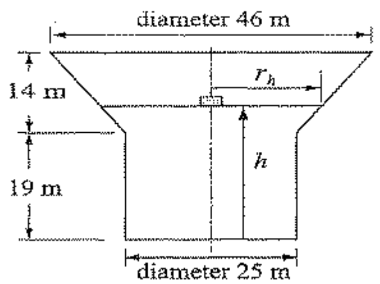

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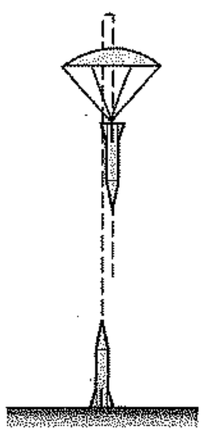

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بسم الله الرحمن الرحيم

KINGDOM OF SAUDI ARABIA
Ministry of Higher Education
Al-Imam Muhammad Ibn Saud Islamic University
College of Science – Department of Mathematics

المملكة العربية السعودية
وزارة التعليم العالي
جامعة الإمام محمد بن سعود الإسلامية
كلية العلوم - قسم الرياضيات

FINAL EXAM - Solutions

SEMESTER: SECOND | **YEAR:** 1430/1431 (2009/2010)

COURSE: STA 111 | **LEVEL:** 3rd & 4th | **SECTIONS:** 174, 175, 176, 177, 191

DATE: 09/07/1431 (21/06/2010) | **DURATION:** 2 HOURS

Instructors: Drs. H. FAIRES & F. BELLALOUNA & A. BERKAOUI & A. S. BEN GHORBAL

Name ____________________ **Section** ______

Student ID ____________ **Your Signature** ____________

	TOTAL MARKS	SCORE
EXERCISE 1	09.00	
EXERCISE 2	15.00	
EXERCISE 3	15.00	
EXERCISE 4	03.00	
TOTAL	42.00	

Solutions

Sample

KINGDOM OF SAUDI ARABIA
Ministry of Higher Education
Al-Imam Muhammad Ibn Saud Islamic University
College of Science – Department of Mathematics

المملكة العربية السعودية
وزارة التعليم العالي
جامعة الإمام محمد بن سعود الإسلامية
كلية العلوم - قسم الرياضيات

INSTRUCTIONS

- Please check that your exam contain **13 Pages** total (including the first page!! & Normal Tables) and **04 EXERC1SES**.
- Please check your test for completeness.
- Read instructions for each problem carefully.
- NO book, NO notes but you may use a calculator which does not graph and which is not programmable.
- Show all your work to get full credit. You must explain how you get your answers using techniques developed in this class so far. Answer with no supporting work, obtained by guess-and-check, or via other methods will result in little or no credit, even correct.
- **Place a box** around **Your Final Answer** to each question.
- Answer the equation in the space provided on the question shcets. If you need more room (space), use the backs of the pages and indicate to the reader that you have done so.
- If you are not sure what a question means, raise your hand and ask me.
- Check your work!

Good Luck!

مع دعائنا لكم بالتوفيق

EXERCISE 1. (09.00 Marks)

PART A (04.50 Marks) A multi-choice exam is composed of **10 questions**; each question consists of **4 possible answers** such that only one is true. The student chooses randomly an answer for each question. Find the probability that:

1. (01.50 Marks) He has exactly 5 correct answers.

Let X be the random variable denoting the number of the correct answers. Thus $X \sim Bin(n = 10, k, p = \frac{1}{4})$

X is Binomial distribution where

$$P(X = k) = \binom{10}{k} \left(\frac{1}{4}\right)^k \times \left(1 - \frac{1}{4}\right)^{10-k}$$

So $P(X = 5) = \binom{10}{5} \times \left(\frac{1}{4}\right)^5 \times \left(\frac{3}{4}\right)^5 = 0.058$

2. (01.50 Marks) He has at most 9 correct answers.

We have

$$\begin{aligned} P(X \leq 9) &= 1 - P(X = 10) \\ &= 1 - \left(\frac{1}{4}\right)^{10} \\ &\simeq 0.999\ldots \simeq 1 \end{aligned}$$

3. (01.50 Marks) He has at least 2 correct answers.

We have

$$\begin{aligned} P(X \geq 2) &= 1 - P(X < 2) \\ &= 1 - P(X \leq 1) \\ &= 1 - P(X = 0) - P(X = 1) \\ &= 1 - \binom{10}{0}\left(\frac{1}{4}\right)^0 \times \left(\frac{3}{4}\right)^{10-0} - \binom{10}{1}\left(\frac{1}{4}\right) \times \left(\frac{3}{4}\right)^9 \\ &= 0.756 \end{aligned}$$

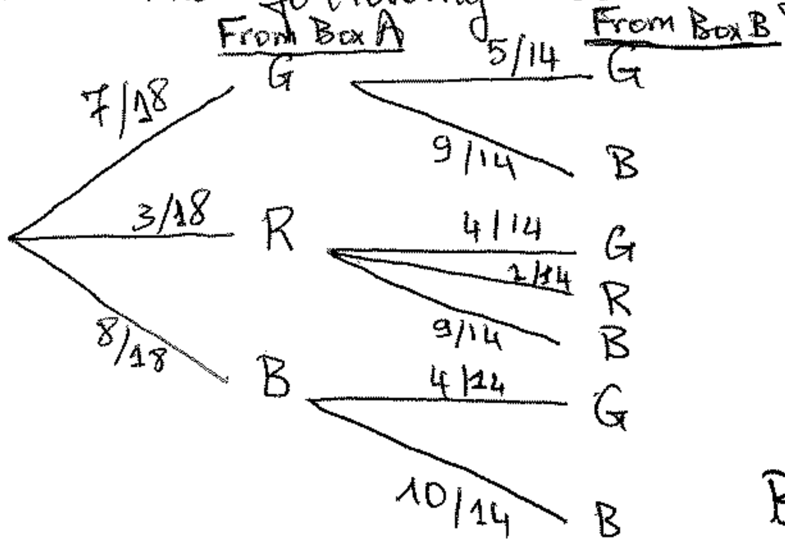
PART B (04.50 Marks) Two boxes A and B are as follows:

- Box A contains 7 green balls, 3 red balls and 8 black balls,
- Box B contains 4 green balls and 9 black balls,

One ball is taken randomly from box *A* (ball 1) and put in box B and then one ball is taken randomly from box *B* (ball 2).

(a) (00.75 Marks) Find the probability that the ball 1 is black.

We have the following tree diagram



B: Black Ball

R: Red Ball

G: Green Ball

Denote:

B_1: Ball 1 is black

B_2: Ball 2 is Black

G_1: Ball 1 is Green

R_2: Ball 1 is Red

Thus $P(\text{Ball 1 is black}) = \dfrac{8}{18}$

(b) (01.50 Marks) Find the probability that the ball 2 is black.

We have

$$P(\text{Ball 2 is black}) = P(B_2 \mid B_1)P(B_1) + P(B_2 \mid G_1)P(G_1) + P(B_2 \mid R_1)P(R_1)$$

$$= \frac{10}{14} \times \frac{8}{18} + \frac{9}{14} \times \frac{7}{18} + \frac{9}{14} \times \frac{3}{18} = \frac{80+63+27}{14 \times 18}$$

$$\simeq 0.675$$

(c) (02.25 Marks) Find the probability that the ball 1 is green, given that the ball 2 is black.

We have

$$P(G_1 \mid B_2) = \frac{P(B_2 \mid G_1)\,P(G_1)}{P(B_2)}$$

$$= \frac{\frac{9}{14} \times 7/18}{\frac{170}{14 \times 18}} = \frac{9 \times 7}{170} \simeq 0.371$$

EXERCISE 2. (15.00 Marks)

PART A (04.50 Marks) The waiting time, in hours, between successive speeders spotted by a radar unit is a continuous random variable X with cumulative distribution function

$$F_X(x) = \begin{cases} 1-e^{-8x} & \text{if } x \geq 0 \\ 0 & \text{if } x < 0 \end{cases}$$

1. (02.00 Marks) Find the density distribution function f_X of the random variable X.

Since F_X is continuous function by intervals then X is a continuous random Variable with probability density function F'_X where

$$F'_X(x) = f_X(x) = \begin{cases} 8e^{-8x} & \text{if } x > 0 \\ 0 & \text{if } x \leq 0 \end{cases}$$

2. (01.00 Marks) Find the probability that the waiting time is lesser than 2 hours, $P(X \leq 2)$.

We have

$$P(X \leq 2) = F_X(2) = 1 - e^{-16} \simeq 1$$

3. (01.50 Marks) Find the probability that the waiting time is bigger than 1 hour, $P(X \geq 1)$.

We have

$$\begin{aligned} P(X \geq 1) &= 1 - P(X < 1) \\ &= 1 - F_X(1) \\ &= 1 - (1 - e^{-8}) = e^{-8} \end{aligned}$$

PART B (03.00 Marks) A continuous random variable X has the following density distribution function

$$f_X(x) = \begin{cases} kx & 0 \le x \le 1 \\ 0 & elsewhere \end{cases}$$

Find the followings:

1. (02.00 Marks) The value of the parameter k.

Since f_X is the p.d.f. of the r.v. X then

$$\int_{-\infty}^{+\infty} f_X(x) = 1 \iff \int_0^1 kx\,dx = 1$$

$$\iff k\left[\frac{x^2}{2}\right]_0^1 = 1$$

$$\iff \frac{k}{2} = 1 \iff k = 2$$

2. (03.00 Marks) The cumulative distribution function F_X, of the random variable X.

We have $f_X(x) = \begin{cases} 2x & \text{if } 0 \le x \le 1 \\ 0 & \text{elsewhere} \end{cases}$

By the definition of c.d.f. we have

$$F_X(x) = \int_{-\infty}^{x} f_X(t)\,dt$$

Then

$$F_X(x) = \begin{cases} 0 & \text{if } x \le 0 \\ x^2 & \text{if } 0 \le x < 1 \\ 1 & \text{if } x \ge 1 \end{cases}$$

3. (01.00 Marks) $P(X \leq 0.5)$,

We have $P(X \leq 0.5) = F_X(0.5)$

$= (0.5)^2$

$= 0.25$

4. (01.50 Marks) $P(-0.4 \leq X \leq 0.4)$,

We have $P(-0.4 \leq X \leq 0.4) = P(X \leq 0.4) - P(X \leq -0.4)$

$= F_X(0.4) - F_X(-0.4)$

$= (0.4)^2 - 0$

$= 0.16$

5. (03.00 Marks) The mean and the variance of X.

X is a continuous r.v.

We have : the mean

$$E[X] = \int_{-\infty}^{+\infty} x f_X(x)\,dx = \int_0^1 2x^2\,dx = \frac{2}{3}$$

We have : the variance

$$V[X] = E[X^2] - (E[X])^2$$

$$= \int_{-\infty}^{+\infty} x^2 f_X(x)\,dx - \left(\frac{2}{3}\right)^2$$

$$= \int_0^1 2x^3\,dx - \frac{4}{9}$$

$$= \frac{1}{2} - \frac{4}{9} = \frac{9-8}{18} = \frac{1}{18}$$

EXERCISE 3. (15.00 Marks)

PART A (04.50 Marks) Let Z be a standard normal random variable and calculate the following probabilities:

(a) (00.50 Mark) $P(Z \leq 1.32)$

We have $P(Z \leq 1.32) = 0.9066$

(b) (01.00 Mark) $P(Z \geq -1.70)$

$$P(Z \geq -1.70) = 1 - P(Z \leq -1.70)$$
$$= 1 - 0.0446$$
$$= 0.9554$$

(c) (01.50 Marks) $P(1.50 \leq Z \leq 2.50)$

We have $P(1.50 \leq Z \leq 2.50) = P(Z \leq 2.50) - P(Z \leq 1.5$
$$= 0.9938 - 0.9332$$
$$= 0.0606$$

(d) (01.50 Marks) $P(|Z| \leq 2.40)$

We have

$$P(|Z| \leq 2.40) = P(-2.40 \leq Z \leq 2.40)$$
$$= P(Z \leq 2.40) - P(Z \leq -2.40$$
$$= 0.9918 - 0.0082$$
$$= 0.9836$$

PART B (06.00 Marks) A continuous random variable X has the normal distribution with mean $\mu = 1$ and standard deviation $\sigma = 2$. Compute the following probabilities by standardizing

a) (01.00 Marks) $P(X \leq 3.5)$,

Since $X \sim \mathcal{N}(\mu = 1, \sigma^2 = 4)$ then we consider the standarized Normal Distribution $Z = \frac{X-\mu}{\sigma} = \frac{X-1}{2} \sim \mathcal{N}(1,0)$

$P(X \leq 3.5) = P\left(Z \leq \frac{3.5-1}{2}\right) = P(Z \leq 1.25) = 0.8944$

b) (01.50 Marks) $P(-2 \leq X \leq 4.4)$

$P(-2 \leq X \leq 4.4) = P\left(\frac{-2-1}{2} \leq Z \leq \frac{4.4-1}{2}\right) = P\left(-\frac{3}{2} \leq Z \leq \frac{3.4}{2}\right)$

$= P(Z \leq 1.70) - P(Z \leq -0.50)$

$= 0.9554 - 0.3085 = 0.6469$

c) (01.50 Marks) $P(X \geq 4.24)$

$P(X \geq 4.24) = 1 - P(X \leq 4.24)$

$= 1 - P\left(Z \leq \frac{4.24-1}{2}\right)$

$= 1 - P(Z \leq 1.62)$

$= 1 - 0.9474 = 0.0526$

d) (02.00 Marks) $P(|X - 8.75| \leq 10)$

$P(|X-8.75| \leq 10) = P(-10 \leq X - 8.75 \leq 10)$

$= P(-1.25 \leq X \leq 18.75)$

$= P\left(\frac{-1.25-1}{2} \leq Z \leq \frac{18.75-1}{2}\right)$

$= P(-1.125 \leq Z \leq 8.875)$

$= P(-1.13 \leq Z \leq 8.88)$

$= P(Z \leq 8.88) - P(Z \leq -1.13)$

$\simeq 1 - 0.1292$

$\simeq 0.8708$

PART C Suppose only 90% of all drivers in Florida regularly wear a seatbelt. A random sample of 100 drivers is selected. Let X be the random variable denoting the number of drivers wearing a seatbelt.

1. (02.00 Marks) What is the probability distribution of the random variable X?

Since any driver will wear a seat belt or not then X is a Binomial Distribution where

$$X \sim Bin(100, 90\%)$$ with $n = 100$ sample size, $p = 0.90$ probability

2. Approximate the probabilities that

(a) (01.50 Marks) Between 89 and 95 of the drivers in the sample regularly wear a seatbelt?

Since $X \sim Bin(100, 0.90)$ with sample size $n = 100 > 30$

then $X \sim \mathcal{N}(\mu_X, \sigma_X^2)$ where $\mu_X = E[X] = np = 90$

The Normal approximation to the Binomial $\sigma_X^2 = npq = 100 \times 0.90 \times 0.1 = 9$

$\sigma_X = 3$

$Z = \frac{X - 90}{3} \sim \mathcal{N}(0,1)$

So

$$P(83 \leq X \leq 95) = P\left(\frac{83-90}{3} \leq Z \leq \frac{95-90}{3}\right)$$
$$= P(-2.33 \leq Z \leq 1.66)$$
$$= P(Z \leq 1.66) - P(Z \leq -2.33) = 0.9515 - 0.0099 = 0.9416$$

(b) (01.00 Marks) Fewer than 93 of those in the sample regularly wear a seatbelt?

$$P(X < 93) = P\left(Z \leq \frac{93-90}{3}\right)$$
$$= P(Z \leq 1.00)$$
$$= 0.8413$$

EXERCISE 4. (03.00 Marks)

Assume that each of your calls to a popular radio station has a probability of 0.02 of connecting, that is, of not obtaining a busy signal. Assume that your calls are independent.

a) (01.50 Marks) What is the probability that your first call that connects is your **tenth call**?

Let X be the random variable denoting the number of calls until the first connection.

So $X \sim Geom(0.02)$: Geometric distribution with parameter $0 < p < 1$

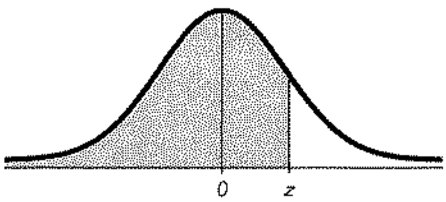
Thus $P(X = k) = (1-p)^{k-1} p$

and

$$P(X=10) = (1-0.02)^9 \times 0.02 = (0.98)^9 \times 0.02$$
$$= 0.0167$$

b) (01.50 Marks) What is the probability that it requires more than **three calls** for you to connect?

$$P(X > 3) = 1 - P(X \leq 2)$$
$$= 1 - \left(P(X=1) + P(X=2)\right)$$
$$= 1 - 0.02 - 0.98 \times 0.02$$
$$= 0.9604$$



POSITIVE z Scores

TABLE A-2 (continued) Cumulative Area from the LEFT

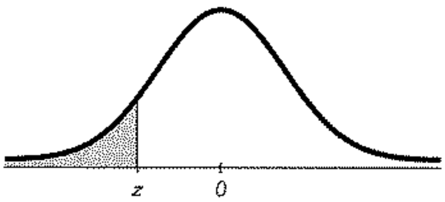
z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495 *	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949 *	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998
3.50 and up	.9999									

NOTE: For values of z above 3.49, use 0.9999 for the area.
*Use these common values that result from interpolation:

z score	Area
1.645	0.9500
2.575	0.9950

Common Critical Values

Confidence Level	Critical Value
0.90	1.645
0.95	1.96
0.99	2.575



NEGATIVE z Scores

TABLE A-2 Standard Normal (z) Distribution: Cumulative Area from the LEFT

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.50 and lower	.0001									
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051 *	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505 *	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
−0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
−0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

NOTE: For values of z below −3.49, use 0.0001 for the area.
*Use these common values that result from interpolation:

z score	Area
−1.645	0.0500
−2.575	0.0050

KINGDOOM OF SAUDI ARABIA
Al-Imam Muhammad Ibn Saud Islamic University
Faculty of Sciences

المملكة العربية السعودية
جامعة الإمام محمد بن سعود الإسلامية
كلية العلوم

Name ______________________________

Section ______ Student ID_________________

Quiz 1 - SOLUTION

8	37	13	24	9	45	23
1	36	18	33	10	12	24
20	22	30	24	47	24	44

1. Construct a stem and leaf plot from the above data.

Answer:

1 8 9 10 11 13 18 20 22 22 24 24 24 24
30 36 36 37 44 45 47

Stem	Leaf						
0	1	8	9				
1	0	1	3	8			
2	0	2	2	4	4	4	4
3	0	6	6	7			
4	4	5	7				

2. Calculate the mean, mode and the quartiles of the data.

Answer:

Mode = 24

Median = 24

Mean = 504 / 21 = 24

Quartile 1 = 12

Quartile 2 = Median = 24

Quartile 3 = 36

Interquartile range (IQR) = 36 – 12 = 24

Upper inner fences = Q3 + 1.5 * IQR = 36 + (1.5 * 24) = 36 + 36 = 72

Lower inner fences = Q1 – (1.5 * IQR) = 12 - (1.5 * 24) = 12 – 36 = - 24

Name ______________________________

Section ______ **Student ID** ________________

Quiz 1 - SOLUTION

8	36	10	2	7	13	17
2	15	28	0	12	1	29
9	29	49	16	2	28	2

1. Construct a stem and leaf plot from the above data.

Answer:

0 1 2 2 2 2 7 8 9 10 12 13 15
16 17 28 28 29 29 36 49

Stem	**Leaf**								
0	0	1	2	2	2	2	7	8	9
1	0	2	3	5	6	7			
2	8	8	9	9					
3	6								
4	9								

2. Calculate the mean, mode and the quartiles of the data.

Answer:

Mode = 2

Median = 12
Mean = 315 / 21 = 15

Quartile 1 = 2

Quartile 2 = Median = 12

Quartile 3 = 28

Interquartile range (IQR) = 28 – 2 = 26

Upper inner fences = Q3 + 1.5 * IQR = 28 + (1.5 * 26) = 28 + 39 = 67

Lower inner fences = Q1 - 1.5 * IQR = 2 - (1.5 * 26) = 2 – 39 = - 37

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MATH 301 - QUIZ 02 – Saturday, December 01, 2007 – Section: 171

SOLUTION

PROBLEM [5 POINTS] Four fair coins are tossed.

1. [1.75 POINT] How many outcomes are in the sample space (list all the elements)?

SOLUTION: Denote by S the sample space, so we have

$$S = \{H,T\}\times\{H,T\}\times\{H,T\}\times\{H,T\}$$
$$= \left\{\begin{matrix} HHHH, HHHT, HHTT, HHTH, HTTT, HTTH, HTHH, HTHT, \\ TTTT, THHH, TTTH, TTHH, THTT, THTH, TTHT, THHT \end{matrix}\right\}$$

Thus, $n(S) = 2^4 = 16$.

2. [1.75 POINT] What is the probability that all four turn up heads?

SOLUTION: Denote by F the event to get all four turn up heads. Thus, $F = \{HHHH\}$ and

$$P(F) = \frac{1}{16}$$

3. [1.50 POINT] Name two events A and B in this sample space where A is a subset of B. Neither of these two events should be the empty event; or the entire sample space, and A and B should not be equal.

SOLUTION: Suppose that A is the event that all four turn up heads and B is the event that the first coin turns up head. Then,

$$A = \{HHHH\},$$
$$B = \{HHHH, HHHT, HHTT, HHTH, HTTT, HTTH, HTHH, HTHT\}.$$

It is obvious to check that A is a subset of B.

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MATH 301 - QUIZ 02 – Sunday, December 02, 2007 – Section: 174

SOLUTION

PROBLEM [5 POINTS] To monitor the performance of a web server the technician in charge records each day whether the server was operational all day (1) or not (0). The technical also records volume of traffic (none, low, medium, high). Consider an "experiment" that consists of recording the performance record of the web server on a particular day. One possible outcome for this experiment is "(1, none)" indicating that the server was operational all day but that there was no traffic.

1. [2 POINTS] Give the sample space (all possible outcomes) for this experiment.

SOLUTION: We have

$$S=\left\{\begin{matrix}(0,\text{none}),(0,\text{low}),(0,\text{medium}),(0,\text{high}),\\(1,\text{none}),(1,\text{low}),(1,\text{medium}),(1,\text{high})\end{matrix}\right\}.$$

2. [1.5 POINTS] Let A be the event that that there was no traffic. Give the outcomes in A.

SOLUTION: We have

$$A=\{(0,\text{none}),(1,\text{none})\}.$$

3. [1.5 POINTS] Let B be the event that the server was operational all day. Give the outcomes in B.

SOLUTION: We have

$$B=\{(1,\text{none}),(1,\text{low}),(1,\text{medium}),(1,\text{high})\}.$$

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MATH 301 – SOLUTION QUIZ 03 – Tuesday, January 08, 2008 – <u>Section:</u> 171, 174

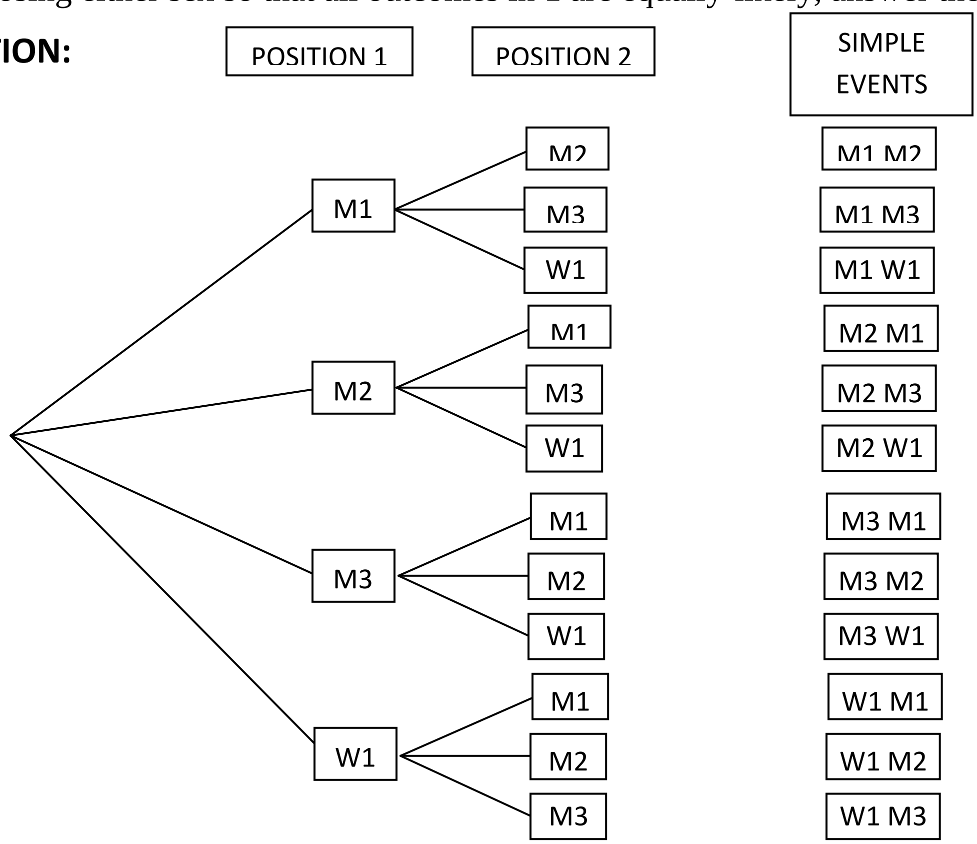
<u>NAME:</u> ____

<u>SIGNATURE:</u> ______________________ ***<u>STUDENT ID:</u>*** _______________

<u>PROBLEM</u>[5 POINTS] A company has 4 applicants for 2 positions: 1 woman and 3 men.

1. [2 POINTS] Use a tree diagram to list all the possible assignments to positions (HINT: Label the applicants as M1, M2, M3, and W1; assume an individual may only hold one position). Supposing that all applicants are qualified and that no preference is given for choosing either sex so that all outcomes in 1 are equally-likely, answer the following**.**

SOLUTION:



2. [1.5 POINTS] Find the probability that two males are chosen to fill the 2 positions.

SOLUTION: P(two males are chosen) = 6/12 or .5

3. [1.5 POINTS] Find the probability that a male is chosen to fill the first position.

SOLUTION: P(a male fills the first position) = 9/12 or .75