

بسم الله الرحمن الرحيم

Kingdom of Saudi Arabia
Ministry of Higher Education
Al-Imam Mohammed Ibn Saud
Islamic University
- College of Science -
Department: Maths
Semester 1. Year: 1432-1433
Duration: 2 hours



المملكة العربية السعودية
وزارة التعليم العالي
جامعة الإمام محمد بن سعود الإسلامية
كلية العلوم
Course Name: Calculus II
Course Code: MAT 102

Final Exam.

Nota bene: This exam contains 2 pages and 6 questions.

Question 1.

Total: [10 Marks]

Evaluate the following integrals.

1. $\int_0^{+\infty} \frac{e^{-x}}{1 + e^{-x}} dx$. [3 Marks]

2. $\int_0^{\frac{\pi}{2}} \sin x \cos^3 x dx$. [3 Marks]

3. $\int_3^4 \frac{x^3 - 2x^2 + x + 3}{x^2 - x - 2} dx$. [4 Marks]

Question 2.

Total: [6 Marks]

Investigate the convergence or divergence of the following improper integrals. Give its values if it converges.

1. $I = \int_0^{\infty} e^{-x} dx$. [3 Marks]

2. $\int_0^1 \frac{1}{\sqrt[3]{x}} dx$, [3 Marks]

Question 3.

Total: [6 Marks]

Investigate the convergence and the absolute convergence of the following series:

1. $\sum_{n \geq 1} (-1)^n \frac{2n + 1}{n^2(3n + 1)}$. [3 Marks]

2. $\sum_{n \geq 1} \left(\frac{2n + 1}{3n + 1} \right)^n$. [3 Marks]

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Question 4.**Total: [7 Marks]**

Let the power series defined by:

$$\sum_{n \geq 1} \frac{1}{2^n \sqrt{n}} (x-1)^n .$$

1. Give the radius of convergence of this power series. [3 Marks]
2. Give its interval of convergence. [4 Marks]

Question 5.**Total: [5 Marks]**

Let f the real function, 2π -periodic, defined by:

$$f(x) = |x| \text{ , if } x \in [-\pi, \pi] .$$

1. Prove that f is an even function. [1 Marks]
2. Give the Fourier expansion of f . [4 Marks]

Question 6.**Total: [6 Marks]**

Let the parametric equation:

$$\begin{cases} x(t) = t^2 + t - 1 \\ y(t) = t^2 - t + 1 \end{cases}$$

1. Give the slope of the tangent line, in every point of its curve. [3 Marks]
2. Give an equation of the tangent line, in the point $(x(0), y(0))$. [3 Marks]