

Mid-term 2

Duration: 1 hour 30 minutes.

Question 1. [1.5+1.5 Marks]

Prove that the following series are convergent and find their sums.

$$a) \sum_{k=1}^{\infty} \frac{2}{(k+1)(k+2)} \qquad b) \sum_{k=2}^{\infty} 2^{-k}$$

Question 2. [1.5+1.5+1.5 Marks]

For each example of the following series, determine if it's convergent or divergent:

$$a) \sum_{n=1}^{\infty} \frac{5n^3 - 4n^2 + 1}{n^4 + 3n^2 - 1} \qquad b) \sum_{n=1}^{\infty} \frac{n!}{2^n} \qquad c) \sum_{n=1}^{\infty} \left(\frac{2n-1}{3n+1} \right)^n$$

Question 3. [1.5+1.5+1.5 Marks]

For each example of the following series, determine if it's absolutely convergent, conditionally convergent or divergent:

$$a) \sum_{n=1}^{\infty} (-1)^n \frac{1}{\sqrt{n+1}} \qquad b) \sum_{n=2}^{\infty} (-1)^n \frac{n}{(n+1)!} \qquad c) \sum_{n=2}^{\infty} (-1)^n e^{-n}$$

Question 4. [2+2 Marks]

Find the radius and the interval of convergence, for each of the two following series:

$$a) \sum_{n=0}^{\infty} \frac{n^2}{2^n} x^n \qquad b) \sum_{n=0}^{\infty} (-1)^n \frac{3^n}{n!} (x-4)^n$$

Question 5. [2+2 Marks]

Give the Taylor-Lagrange expansion of the two functions:

$$a) f(x) = \frac{1}{1+2x} \qquad b) g(x) = \ln(1+2x)$$

Specify the interval of convergence of this two series.