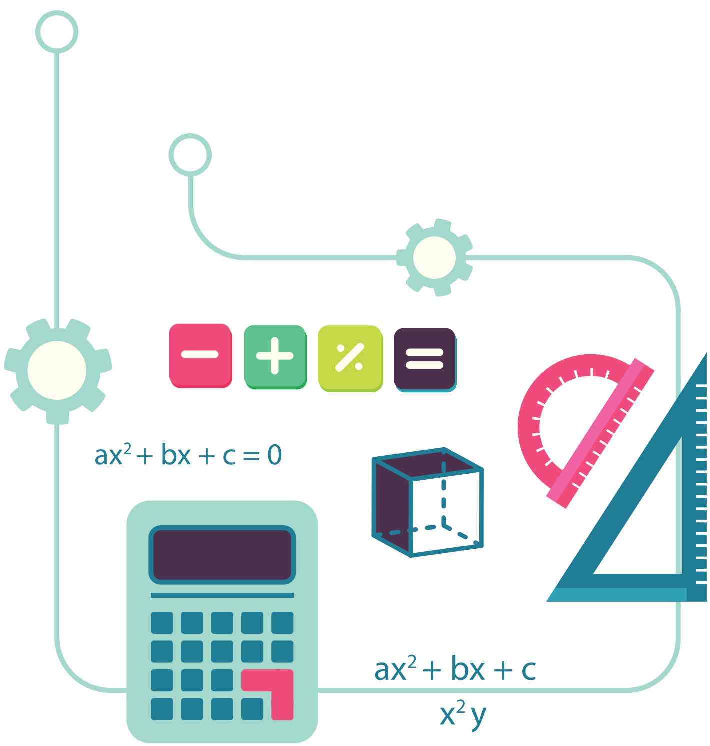
المركز الوطني للقياس
National Center for Assessment

هيئة تقويم التعليم والتدريب
Education & Training Evaluation Commission

نسخة تجريبية

LEARNING OUTCOMES

MATHMATICS



بسم الله الرحمن الرحيم

مخرجات التعلم

تخصص الرياضيات

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Introduction :

Higher Education in Saudi Arabia has witnessed a rapid development in the recent years, through inaugurating new public and private universities around the country. However, this may have an impact upon the teaching system in general and program outcomes in particular. Therefore, the Ministry of Education has endeavored to improve the quality of program outcomes in all Saudi universities. It then launched the project of learning outcomes (LOs) in Higher Education, in collaboration with the National Center for Assessment. The Bologna process which focuses primarily on LOs has been adopted widely, particularly in most European countries. Thus, this promising project will draw on the Bologna process to come up with LOs for academic programs that are being taught in Saudi universities.

LOs are basically used to ensure the quality of learning and teaching. By using them, it becomes easier to compare two different programs of the same major (i.e. benchmarking). They also help academic departments and teachers to develop course materials and determine course objectives. More importantly, they play a key role in linking teaching and learning with assessment and assisting academic programs to gain accreditation.

Furthermore, LOs have some benefits for students (stakeholders). They will provide them with the necessary information of the program they would like to join. In other words, LOs help stakeholders to know what kind of achievement they will gain by completing a certain program in cognitive (essential knowledge), behavioral (skills and abilities) and affective (attitudes, values or beliefs) domains.

Stages of the project

This project has gone through various stages, as illustrated in Figure 1. It began with forming the main committees that will participate in this project. The National Center for Assessment ran workshops on how to write LOs and exam items based on LOs in which faculty members from various Saudi universities participated. Here are the main stages in more details.

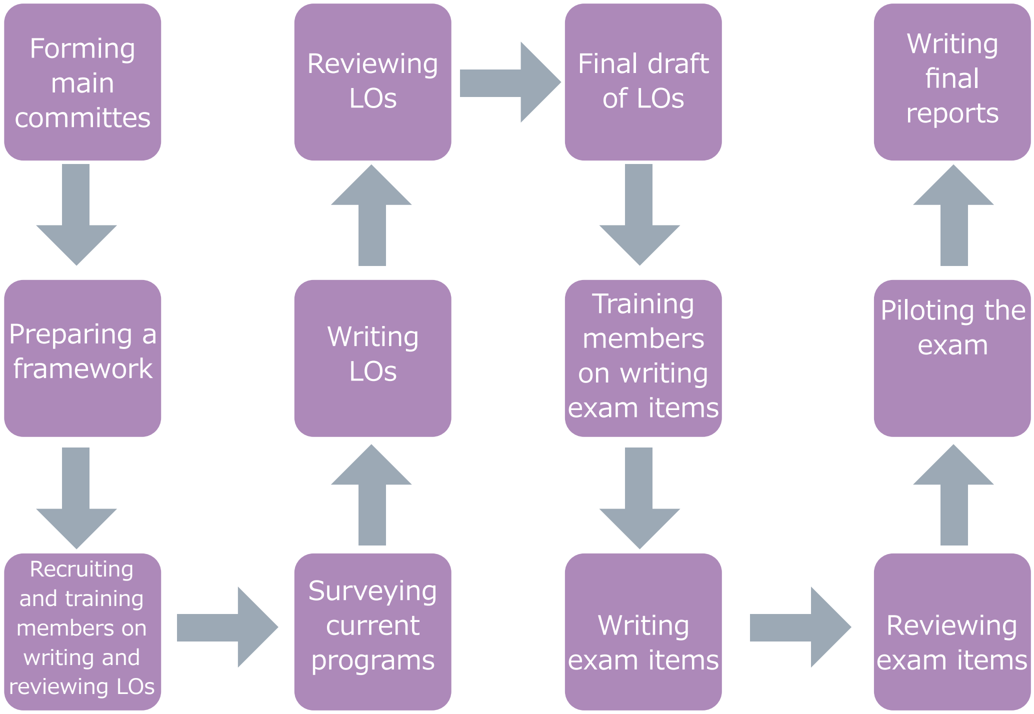

Figure 1. Stages of the LOs project

First Phase: Surveying current academic programs

This phase aims to survey the content of national and international academic programs. The objective is to establish the LOs based on these programs and identify the extent of coverage of these LOs in the academic programs in Saudi universities. The most important steps of this phase include:

1.1 Identifying LOs

A comprehensive survey has been conducted on all programs of Mathematics in Saudi universities, in an attempt to identify the LOs of this major.

1.2 Analyzing the content of the national programs

After collecting After collecting the content of relevant programs, a thorough analysis was done in order to identify the common components in these programs and the ones that are unique to certain programs. This procedure includes the following:

- Identifying the main components of the major in all Saudi universities.
- Determining the percentages of the main components in these programs.
- Identifying the common sub-components in these programs.
- Determining the percentages of the sub-components in these programs.

1.3 Analyzing the content of some international programs:

The previous procedure was done on the programs of the following universities:

- Connecticut University.
- Penn State University.
- University of Texas-San Antonio.

1.4 Comparing the content of the national and international programs

A comparison was made among the components of the national and international programs in order to identify the common main and sub-components in these programs and the ones that are unique to certain programs.

Second Phase: Proposing the LOs of the program

This phase focuses mainly on identifying the components and their importance in the program. This procedure includes the following:

1. Defining the major accurately and comprehensively in order to determine the features that distinguish it from other similar programs.
2. Proposing the components of the program, based on the survey in the previous phase, and identifying the programs to which they are compared for benchmarking purposes.
3. Determining the importance of each component. To do so, the teaching hours of each component in the program have been calculated.
4. Dividing the main components into sub-components.
5. Identifying the importance of the sub-components, as is illustrated in Table 1.
6. Defining the main components and sub-components of the program on which the LOs will be based.

Table 1. Percentages of main components and sub-components of Mathematics

Main component	%	Sub-component	%
Logic and Foundation	7.15	Foundation of Mathematics	7.15
Algebra	23.04	Linear Algebra	7.15
		Number Theory	4.08
		Group Theory	7.15
		Rings and Fields	4.66
Analysis	14.3	Real Analysis	7.15
		Complex Analysis	7.15
Topology and Geometry	9.53	Topology	4.87
		Differential Geometry	4.66
Differentiation and Integration	28.6	Differentiation	7.15
		Integration	7.15
		Calculus in Several Variables	7.15
		Ordinary Differential Equations	7.15
Statistics	7.15	Statistics and Probability	7.15
Applied Mathematics	10.23	Numerical Analysis	6.37
		Partial Differential Equations	3.86
Total			100

Third Phase: Writing LOs

When writing LOs, the following points have been taken into consideration:

1. Drawing on the criteria of writing LOs reported in the literature, e.g. using measurable verbs.

2. Covering Bloom's Taxonomy levels, particularly knowledge, application and analysis.

3. Determining the target content, taking into account the division of the program (i.e. main components, sub-components and LOs) as well as the identification importance of program main and sub-components.

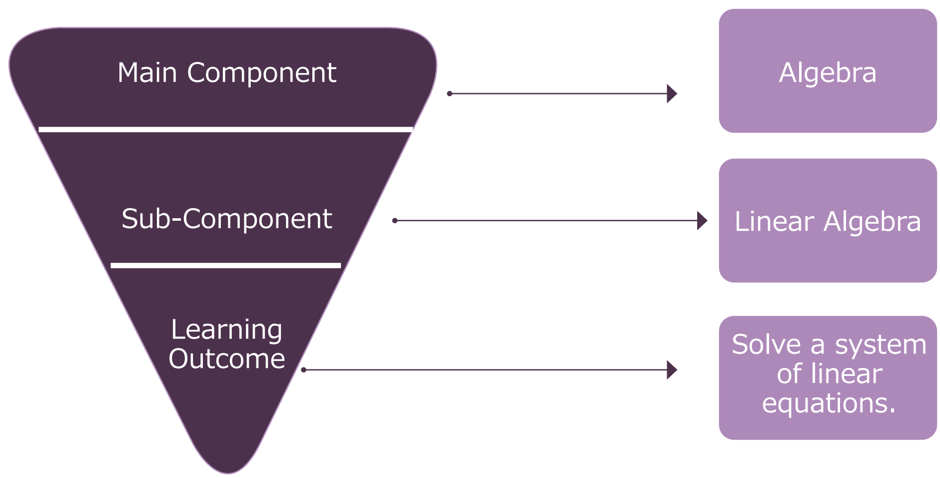

Figure 2. Illustration of the program division into main components, sub-components and LOs

Fourth Phase: Reviewing LOs :

To ensure the quality of the writing process and the use of criteria of writing LOs, the review process went through three stages:

1. Program experts

Three experts of the program were recruited for reviewing the LOs. They were trained on how to assess LOs.

2. National universities

A draft of the LOs was sent to all Saudi universities, in an attempt to get feedback from the faculty members of Mathematics Department in these universities. This was a very crucial step as it showed us to what extent the LOs covered the major and whether the importance of main components and sub-components was determined properly.

3. Electronic review

The draft of the LOs was also posted on the website of the National Center for Assessment, in an attempt to get feedback from experts of Mathematics everywhere. Then, it was advertised that the LOs for Mathematics were available online for review .

Fifth Phase: Revising LOs :

The comments and feedback received from the review process were approved by the reviewing committee and then sent to the committee of writing LOs to revise them accordingly. After revising the LOs, the reviewing committee approved the changes that were made:

Sixth Phase: Final draft of LOs:

After the revision process, the final draft of the LOs for Mathematics was written for official use in the future, as is shown in Table 2.

1-Main Component: Logic and Foundation

Description: Graduates are expected to understand the logical basis of mathematics and basic concepts of various fields of mathematics.

Sub-Components	Learning Outcomes
1.1 Foundation of Mathematics : To comprehend mathematical logic, methods of proof, set theory, binary relations, binary operations, morphisms, groups, rings and fields.	1) Develop communication skills necessary for reading and writing mathematical proofs.
	2) Demonstrate knowledge of the basic theory of sets, functions, relations, modular arithmetic and common sets of numbers.
	3) Generate and criticize written proofs using standard methods such as direct, contradiction, and induction.

2-Main Component: Algebra

Graduates are expected to grasp the main results about matrices, systems of linear equations, vector spaces, inner product spaces, linear transformations, and diagonalization.

Sub-Components	Learning Outcomes
2.1 Linear Algebra To grasp properties of matrices and their operations, systems of linear equations, vector spaces, inner product spaces, dual spaces, bilinear forms, polynomials and matrices, triangulation of matrices, linear transformations, Hamilton-Cayley theorem, eigenvalues, eigenvectors and diagonalization.	1) Calculate matrix operations (addition, multiplication, inverse, transpose, trace, determinant, elementary operations) and prove their main properties.
	2) Solve a system of linear equations.
	3) Demonstrate knowledge of the theory of vector spaces, its maim concepts and results.
	4) Recognize and prove the main properties of inner product and inner product spaces including Cauchy Schwarz inequality.
	5) Apply Gram-Schmidt process to find an orthonormal basis.
	6) Determine the range, kernel and matrix representations of a given linear transformation.
	7) Compute the characteristic polynomial, eigenvalues and eigenvectors of a matrix and identify diagonalizabity.
2.2 Number Theory: To grasp divisibility of integers, Euclidean algorithm, prime numbers and their properties, linear Diophantine equations, congruences, Chinese remainder theorem, Fermat's little theorem, Euler's theorem, Wilson's theorem, arithmetic functions, and Pythagorean triples..	1) Demonstrate knowledge of divisibility, prime numbers and the Euclidean Algorithm.
	2) Solve linear Diophantine equations and congruences of various types, and use the theory of congruences in applications.
	3) Prove and apply properties of multiplicative functions such as the Euler phi-function.
	4) Prove Wilson's Theorem and Fermat's Little theorem and use them in applications.

Sub-Components	Learning Outcomes
2.3 Group Theory: To be familiar with the main properties of groups, group homomorphisms, simple groups, permutation groups, a group of action on a set, p-groups, Cauchy’s theorem, Sylow theorems, external and internal direct products of groups, dihedral groups, quaternions, and groups of automorphisms of cyclic groups.	1) Define and create examples of groups, group homomorphisms, isomorphisms, abelian groups, cyclic groups, normal subgroups, quotient groups, and group actions.
	2) Produce the proofs of main theorems and key results of groups.
	3) Verify group axioms in examples.
	4) Derive and apply Lagrange's theorem, the isomorphism theorems, and Cayley's theorem.
	5) Demonstrate knowledge of external and internal direct products of groups and groups of automorphisms of cyclic groups.
	6) Define and create examples of simple groups, permutation groups, dihedral groups and quaternions
	7) Prove Sylow theorems and apply them to classify groups.

Sub-Components	Learning Outcomes
2.4 Rings and Fields: To understand the main theorems and properties of rings including ideals and factor rings, integral and principal rings, field of quotients, direct sum of rings, modules, Euclidean rings, ring of polynomials, field extensions, splitting fields, and finite fields.	1) Define and create examples of rings, ideals, factor rings, ring homomorphisms, integral domains, direct sum of rings, and characteristic of a ring.
	2) Recognize the main properties of principal rings, Euclidean rings, and ring of polynomial.
	3) Demonstrate knowledge of fields, field extensions, and splitting fields.
	4) Define and create examples of modules.
	5) Establish the proofs of main theorems and key results of rings and fields.
	6) Verify the satisfactory of axioms of rings and fields in specific examples.
	7) Apply the main theorems of rings and fields to deduce some basic results.
	8) Recognize the basic properties and examples of finite fields.

3.Main Component: Analysis

Graduates are expected to understand theories of sequences and series, limits of functions, continuity, differentiation, all in the context of real and complex variables, sequence and series of functions, measure, and Reimann and Lebesgue integrations.

Sub-Components	Learning Outcomes
3.1 Real Analysis: To be familiar with the basic theoretical and topological properties of the field of real numbers, sequences, limit of a function, continuous functions, uniform continuity, the derivative of a function, mean value theorem, Taylor theorem, Riemann integration, sequences and series of functions, measure spaces, Lebesgue measure, Lebesgue integration, and the main convergence theorems.	1) Describe the real line as a complete, ordered field.
	2) Determine the main topological properties of subsets of the real line.
	3) Use the various basic definitions and properties of sequence convergence for numbers and functions.
	4) Derive the continuity and differentiability of real valued functions.
	5) Prove and apply the mean value theorem and the fundamental theorem of calculus.
	6) Prove various results arising in the context of real analysis.
	7) Demonstrate knowledge of Riemann integration of a real-valued function and its main properties, and prove the basic properties of Riemann integration.
	8) Demonstrate knowledge of measure spaces, Lebesgue measure, Lebesgue integration, and the main convergence theorems.

Sub-Components	Learning Outcomes
3.2 Complex Analysis: To grasp the basic properties of complex numbers, limits and continuity of complex functions, analytic functions, Cauchy-Riemann equations, harmonic functions, exponential, trigonometric, hyperbolic functions and logarithmic functions, complex integration, contour integrals, Cauchy's theorem, Cauchy's formula, bounds on analytic functions, series representation of analytic functions, Taylor and Laurent series, power series, zeros and singularities, residue theory, and applications to real and improper integrals.	1) Describe the geometric and algebraic representations of complex numbers.
	2) Define and investigate limits, continuity, and consequences of continuity for complex valued functions.
	3) Apply main results and concepts of analyticity, harmonic and entire functions, and Cauchy-Riemann equations.
	4) Investigate sequences and series of analytic functions and types of convergence.
	5) Compute complex contour integrals directly and compute them by the fundamental theorem.
	6) Apply the various forms of Cauchy integral theorem and the Cauchy integral formula.
	7) Describe Taylor and Laurent series representations of functions.
	8) Classify singularities and poles of a complex function, and compute the residues.
	9) Apply the residue theorem to evaluate complex integrals.

4.Main Component: Topology and Geometry

Description: Graduates are expected to explain the concepts of differential geometry, in addition to the generic form of geometry known as topology. The student should also master the properties of regular shapes and manifolds.

Sub-Components	Learning Outcomes
4.1 Topology: To understand metric spaces, topological spaces, bases, open and closed sets, homeomorphisms, product of spaces, open and closed maps, metric spaces, connectedness, compactness, separation axioms, Lindelof spaces and the first and second axioms of countability.	1) Describe topological spaces and continuous functions.
	2) Explain the concept of metric spaces and distinguish the compactness types in metric spaces.
	3) Recognize and prove the homeomorphism properties in theorems and examples.
	4) Prove essential theorems concerning topological spaces, continuous functions, and product topologies.
	5) Demonstrate knowledge of the concepts of separation axioms and Lindelof spaces.
	6) Summarize the concepts of connectedness and compactness, and then prove related results.
	7) Describe various examples distinguishing general, geometric, and algebraic topology.
	8) Explain the countability axioms.

Sub-Components	Learning Outcomes
4.2 Differential Geometry: To draw curves and surfaces in plane and space with various notions of curvature using exterior differential calculus, and to demonstrate knowledge of Riemannian geometry in higher dimensions concentrating in three main parts: curves and surfaces, geodesic and curvature and manifolds.	1) Parameterize curves and find arc length.
	2) Derive Frenet formulae.
	3) Use the local canonical form and corresponding curves.
	4) Distinguish types of curvature and torsion.
	5) Parameterize surfaces and recognize regular surfaces and regular values.
	6) Use first and second fundamental forms.
	7) Analyze geodesic lines and curves.
	8) Define manifolds and use Gauss map and its properties.

5.Main Component: Differentiation and Integrations

Graduates are expected to compute limits, continuity, derivative of functions and vectors. The graduate should also master differentiation, integration, advanced calculus and differential equations.

Sub-Components	Learning Outcomes
5.1 Differentiation: To represent functions and their graphs, operations on functions, limits, continuity, derivatives, applications of differentiation for plotting functions, maxima and minima, and solving natural and physical problems.	1) Define a function, its properties, and continuity and draw its graph.
	2) Compute one-sided limits, limits, infinite limits, and limits at infinity.
	3) List the rules of derivatives of the basic functions.
	4) Determine critical points and extreme values of functions and draw their graphs by monotonicity and convexity.
	5) Calculate derivatives of functions using its formal definition.
	6) Perform implicit differentiation.
	7) Apply intermediate and mean value theorems.
	8) Apply L'Hôpital's Rule to compute indeterminate form limits.
	9) Solve natural problems as applications to differentiation.

Sub-Components	Learning Outcomes
5.2 Integration To know how to compute anti-derivatives, indefinite integral and its properties, Riemann sum, fundamental theorem of calculus, and integration techniques. The graduate is also expected to master applications of definite integrals in geometry, infinite sequences, infinite series including Taylor and MacLaurin series.	1) Find anti-derivatives of suitable functions.
	2) Apply the fundamental theorem of calculus.
	3) Differentiate and integrate the exponential and logarithmic functions, and use them to model growth and decay.
	4) Select the suitable method to solve integrals containing trigonometric, inverse trigonometric and hyperbolic functions.
	5) Determine convergence or divergence of sequences and series.
	6) Represent suitable functions using Taylor and MacLaurin series.
	7) Define and evaluate an improper integral.
	8) Use integration to solve application problems including finding arc length, area, and volume.

Sub-Components	Learning Outcomes
5.3 Calculus in Several Variables To explain conic sections, polar coordinates, vectors in space, dot and cross products, lines and planes in space, curves and surfaces in space, cylindrical and spherical coordinates, functions of several variables, partial and total derivatives, directional derivative and gradient, maxima and minima, Lagrange multiplier, multiple integrals and their applications..	1) Recognize the types of conic sections.
	2) Illustrate with examples the vectors in the plane and in a three dimensional Euclidean space.
	3) Compute the dot and cross products in different types of coordinates including cylindrical and spherical coordinates.
	4) Define vector-valued functions and their differentiation and integration.
	5) Explain functions of several variables, limits, continuity, partial derivatives, the chain rule, directional derivatives, gradients, tangent planes, normal lines, and extrema of functions of two variables with Lagrange multipliers.
	6) Calculate double integrals, triple integrals in cylindrical and spherical coordinates, and change of variables in multiple integrals.
	7) Master the main techniques of vector analysis to handle vector fields, line integrals and Green's theorem.
	8) Perform a number of applications in geometry, nature and physics.

Sub-Components	Learning Outcomes
5.4 Ordinary Differential Equations To solve first order differential equations (ODEs), second order linear ODEs, homogeneous non-homogeneous linear ODEs, and higher order linear ODEs, by using variation of parameters method. The graduate is also expected to master Laplace transform, first and second shifting theorems, Dirac function, convolution and solving initial value problems, as well as advanced methods for solving linear and some nonlinear ODEs.	1) Describe the concept, meaning and solutions of ODEs.
	2) Solve ODEs by various methods including separation of variables, transformation of variables, exactness, and method of inspection.
	3) Determine complimentary solution, auxiliary equation, linear independence, and Wronskians for linear ODEs.
	4) Solve ODEs by undetermined coefficients and variation of parameters methods
	5) Apply ODEs to problems in engineering and related fields.
	6) Solve ODEs using various methods such as power series, and Laplace transform.
	7) Solve some of the famous nonlinear ODEs and systems of linear ODEs.

6. Main Component: Statistics.

Graduates are expected to analyze data using a variety of tools, various situations of probability either axiomatically or computationally and be able to apply theoretical statistics.

Sub-Components	Learning Outcomes
6.1 Statistics and Probability To draw conclusions about probability axioms, counting rules, random variables, discrete distributions, continuous distributions with computing means, expectation, and standard deviation. The graduate should also be able to know how to test hypothesis and moment, and find generating functions, correlation and regression.	1) Present and interpret statistical data, both numerically and graphically including correlation and regression.
	2) Describe discrete data graphically and compute measures of centrality and dispersion.
	3) Derive laws of skewness and kurtosis using moment generating functions.
	4) Show the probability axioms, and prove the main probability theorems.
	5) Calculate probabilities using modeling sample spaces and applying rules of permutations and combinations, additive and multiplicative laws, and conditional probability.
	6) Construct the probability distribution of a random variable based on a real-world situation, and use it to compute expectation and variance.
	7) Compute probabilities based on practical situations using the binomial and normal distributions.
	8) Use the normal distribution to test statistical hypotheses and compute confidence intervals.

7. Main Component: Applied Mathematics.

Graduates are expected to solve numerically linear and non-linear system of equations, and ordinary differential equations. The student should also demonstrate knowledge of numerical differentiation and integration, classification of partial differential equations and their solutions.

Sub-Components	Learning Outcomes
7.1 Numerical Analysis To employ numerical methods including error estimation in order to solve linear and non-linear system of equations, ordinary differential equations, boundary value problems in ordinary differential equations, numerical differentiation and integration, interpolation, and polynomial approximation.	1) Compute errors and understand error estimation.
	2) Solve equations in one unknown real variable using bisection, fixed point, and Newton's methods and realize the length of these methods to converge to an exact solution.
	3) Employ suitable methods to solve nonlinear equations numerically.
	4) Generate the approximate solution to a system of ordinary differential equations.
	5) Construct an interpolation polynomial using either Lagrange or Newton formula.
	6) Derive formulas to approximate the derivative of a function.
	7) Apply Trapezoidal and Simpson rules to approximate an integral.

Sub-Components	Learning Outcomes
7.2 Partial Differential Equations To classify partial differential equations (PDEs), memorize Lagrange's method for solving quasi-linear equation, and find its solution by separation of variables. The graduate is also expected to derive solutions for boundary value problems by Fourier, Laplace transforms and Green function, and wave and heat equations in one dimension.	1) Classify PDEs according to order and linearity.
	2) Describe the concept PDEs, their solutions and their applications.
	3) Apply suitable methods to solve some first and second PDEs.
	4) Distinguish second order PDEs as elliptic– parabolic-hyperbolic, describe their characteristic properties and solve them.
	5) Apply Green's function to solve appropriate problems.
	6) Demonstrate accurate and efficient use of Fourier and Laplace transforms and their applications in the theory of PDEs.
	7) Solve Laplace, wave, and heat equations