





## **Course Specification**

— (Postgraduate Programs )

**Course Title**: Probability Theory

Course Code: MAT 7101

**Program: Doctor of Philosophy in Mathematics** 

**Department: Mathematics and Statistics** 

College: Science

Institution: Imam Mohammad Ibn Saud Islamic University

Version: 2024 - V1

Last Revision Date: None

## **Table of Contents**

| A. General information about the course:                                       | 3 |
|--|---|
| B. Course Learning Outcomes (CLOs), Teaching Strategies and Assessment Methods | 4 |
| C. Course Content  | 5 |
| D. Students Assessment Activities  | 5 |
| E. Learning Resources and Facilities   | 6 |
| F. Assessment of Course Quality  | 6 |
| G. Specification Approval  | 7 |





| A. | General | inf | formati | ion a | bout | the | course: |
|----|---------|-----|---------|-------|------|-----|---------|
|----|---------|-----|---------|-------|------|-----|---------|

### 1. Course Identification

| 1. C  | 1. Credit hours:      |                   |                  |               |          |
|---|-----------------------|-------------------|------------------|---------------|----------|
| 4 (4 I  | Lectures, 0 Lab, 0 Tu | torial)           |                  |               |          |
| 2. C  | ourse type            |                   |                  |               |          |
| A.  | $\square$ University  | ☐ College         | □ Program        | ☐ Track       | ☐ Others |
| В.  | oxtimes Required      |                   | □ Ele            | ctive         |          |
| 3. L  | evel/Year at wh       | nich this course  | is offered: Leve | el 2 / Year 1 |          |
| 4. Course general Description:  |                       |                   |                  |               |          |
| The course covers the basic principles of the measure-theoretic probability theory. Topics include the axioms of probability, random variables and vectors, independence of events and variables; modes of probabilistic convergence, laws of large numbers; characteristic functions, the central limit theorem, conditional probability and expectation, introduction to martingales. |                       |                   |                  |               |          |
| E D   | HO HORILINOPPORT      | o for this course |                  |               |          |

#### 5. Pre-requirements for this course (if any):

None.

## 6. Co-requisites for this course (if any):

None.

### 7. Course Main Objective(s):

The objective of this course is first to understand general concepts in measure-theoretic probability theory: random variable, independence, and different type of convergence, laws of large numbers, and the Central Limit Theorem. As application, limit theorems are used to approximate probabilities of average and sums of independent identically-distributed random variables. Finally the conditional expectation with respect to a sigma-field is introduced and applied to martingales.

### 2. Teaching mode (mark all that apply)

| No | Mode of Instruction   | Contact Hours | Percentage |
|----|---|---------------|------------|
| 1  | Traditional classroom   | 60            | 100%       |
| 2  | E-learning  | 0             | 0%         |
| 3  | <ul><li>Hybrid</li><li>Traditional classroom</li><li>E-learning</li></ul> | 0             | 0%         |
| 4  | Distance learning   | 0             | 0%         |

### 3. Contact Hours (based on the academic semester)

| No | Activity | Contact Hours |
|----|----------|---------------|
| 1. | Lectures | 60            |





| 2. | Laboratory/Studio | 0  |
|----|-------------------|----|
| 3. | Field             | 0  |
| 4. | Tutorial          | 0  |
| 5. | Others (specify)  | 0  |
|    | Total             | 60 |

# B. Course Learning Outcomes (CLOs), Teaching Strategies and Assessment Methods

| Code | Course Learning Outcomes   | Code of CLOs aligned with program | Teaching<br>Strategies              | Assessment<br>Methods                      |
|------|--|-----------------------------------|-------------------------------------|--|
| 1.0  | Knowledge and under  | standing                          |                                     |  |
| 1.1  | To list the different types of convergences, the laws of large numbers, and the central limit theorem. | K1, K2                            | 4 lecture hours\week                | Direct: Regular<br>Exams                   |
| 1.2  | To reproduce the notions of conditional expectation and martingale                                     | K1, K2                            | • 4 lecture hours\week • Self-study | Direct: Short<br>Quizzes                   |
| 2.0  | Skills   |                                   |                                     |  |
| 2.1  | To develop techniques of proof in probability theory.  | S1, S2                            | Self-study                          | Direct:<br>Participations<br>Short Quizzes |
| 2.2  | To develop oral communication and technical writing skills through random variables.                   | 83                                | Real-life problems                  | Direct:<br>Homework and<br>Mini projects   |
| 2.3  | To use Internet in searching for different types of modes of convergences.                             | <b>S4</b>                         | Real-life problems                  | Direct: Short<br>Quizzes                   |
| 2.4  | To write out deep proofs in applications of Central Limit Theorem.                                     | S1, S2                            | Self-study                          | Direct:<br>Participations                  |

| Code 3.0 | Course Learning Outcomes Values, autonomy, and                        | Code of CLOs aligned<br>with program<br>d responsibility | Teaching<br>Strategies          | Assessment<br>Methods                    |
|----------|---|--|---------------------------------|--|
| 3.1      | To execute assignments and homework with independence responsibility. | V1, V3   | Personal questions              | Direct:<br>Participation                 |
| 3.2      | To cooperate with team works.   | V1, V2   | Teamwork and class discussions. | Direct:<br>Homework and<br>Mini projects |

### **C.** Course Content

| No | List of Topics   | Contact Hours |
|----|--|---------------|
| 1. | <b>Foundations of Probability:</b> Probability Space and Random Variables, Distribution Function of a Random variable, Random Elements, Vectors and Joint Distributions, Expectation and Moments, Inequalities for Moments and Probabilities.  | 12            |
| 2. | <b>Independence:</b> Independent Events and Classes, Independent Random Elements, Independent Random Variables, Addition of Independent Random variables, Borel–Cantelli Lemma and zero-one Law.   | 12            |
| 3. | <b>Convergence and Related Topics:</b> Modes of Probabilistic Convergence: Almost Sure Convergence, Convergence in Probability, Convergence in Order Mean, Convergence in Distribution, Relationships Between Forms of Convergence, Series of Independent Random Variables, Laws of Large Numbers. | 12            |
| 4. | <b>Characteristic Functions and Central Limit Theorem:</b> Definition and Simple Properties, Characteristic Function and Moments, Inversion and Uniqueness, Continuity Theorem for Characteristic Functions, Some Applications (Central Limit Theorem).  | 12            |
| 5. | <b>Conditioning and Martingales:</b> Conditional Expectation Given a Sigma-Field, Conditional Probability Given a Sigma-Field, Definition and Basic Properties of Martingales  | 12            |
|    | Total  | 60            |

### **D. Students Assessment Activities**

| No | Assessment Activities *           | Assessment timing (in week no) | Percentage of Total Assessment Score |
|----|-----------------------------------|--------------------------------|--------------------------------------|
| 1. | HomeWorks, Quizzes, Mini projects | <b>During the semester</b>     | 30%                                  |
| 2. | Midterm                           | Week 9-10                      | 30%                                  |





| No | Assessment Activities * | Assessment timing (in week no) | Percentage of Total Assessment Score |
|----|-------------------------|--------------------------------|--------------------------------------|
| 3. | Final Exam              | Week 15-16                     | 40%                                  |

<sup>\*</sup>Assessment Activities (i.e., Written test, oral test, oral presentation, group project, essay, etc.).

## **E.** Learning Resources and Facilities

### 1. References and Learning Resources

| Essential References     | R. Leadbetter et al., A Basic Course in Measure and Probability, Theory for Applications, Cambridge University Press 2014. (Main Reference)  |  |  |
|--------------------------|--|--|--|
| Supportive References    | <ul> <li>R G. Tucker, A Graduate Course in Probability, Probability and Mathematical Statistics, Dover publications, 2014.</li> <li>E. Cınlar, Probability and Stochastics, Graduate Texts in Mathematics, 261, Springer, 2011.</li> </ul> |  |  |
| Electronic Materials     | None   |  |  |
| Other Learning Materials | None   |  |  |

## 2. Educational and Research Facilities and Equipment Required:

| Items   | Resources   |
|---|---|
| facilities (Classrooms, laboratories, exhibition rooms, simulation rooms, etc.) | <ul> <li>Each class room should be equipped with a whiteboard and a projector.</li> <li>Laboratories should be equipped with computers and an internet connection.</li> </ul> |
| Technology equipment (projector, smart board, software)                         | The rooms should be equipped with data show and Smart Board.  |
| Other equipment (depending on the nature of the specialty)                      | None  |

## F. Assessment of Course Quality

| Assessment Areas/Issues                 | Assessor   | Assessment Methods   |
|---|------------|--|
| Effectiveness of teaching               | Students   | During the semester and at the end of the course each student will complete two evaluation forms.  |
| Effectiveness of<br>Students assessment | Instructor | At the end of each semester the course instructor should complete the course report, including a summary of student questionnaire responses appraising progress and identifying changes that |





| Assessment Areas/Issues                     | Assessor   | Assessment Methods   |
|---|------------|--|
|   |            | need to be made if necessary.  |
| Quality of learning resources               | Students   | During the semester and at the end of the course each student will complete two evaluation forms.  |
| The extent to which CLOs have been achieved | Instructor | At the end of each semester the course instructor should complete the course report, including a summary of student questionnaire responses appraising progress and identifying changes that need to be made if necessary. |
| Other                                       | None       |  |

Assessors (Students, Faculty, Program Leaders, Peer Reviewer, Others (specify)
Assessment Methods (Direct, Indirect)

## **G. Specification Approval**

| COUNCIL /COMMITTEE | MATHEMATICS AND STATISTICS DEPARTMENT COUNCIL |
|--------------------|---|
| REFERENCE NO.      | 8/1446  |
| DATE               | 05/04/1446 (08/10/2024)                       |

