Course Specification

(Postgraduate Programs)

Course Title:	Measure and Integration
Course Code:	MAT 7111
Program:	Doctor of Philosophy in Mathematics
Department:	Mathematics and Statistics
College:	Science
Institution:	Imam Mohammad Ibn Saud Islamic University
Version:	2024 – V1
Last Revision Date:	None

Table of Contents

A. General information about the course:

1. Course Identification

1. Credit hours:					
4 (4 Lectures, 0 Lab, 0 Tutorial)					
2. Course type					
A.	☐ University	☐ College	☒ Program	☐ Track	☐ Others
B.	☒ Required			☐ Elective	
3. Level/Year at which this course is offered: Level 1 / Year 1					
4. Course general Description:					
This course describes the most important ideas, basics of measure and integration theories, and proofs of the fundamental theorems underlying the theory of measure and integration. First, we introduce Caratheodory Theorems and the construction of outer measure. Integration theory is presented in its abstract setting. Properties of measurable functions and integrable functions are given, leading to the definition of L1 space and its properties. Then the main convergence theorems and their applications are proved. Starting from basic inequalities, the Banach space Lp is introduced. The Riesz Representation Theorem and dual of Lp are given. Then the concepts of density and separability are related to approximation theorems. Weak convergence and main properties are also introduced.					
5. Pre-requirements for this course (if any)**:**					
None.					
6. Co-requisites for this course (if any)**:**					
None.					
7. Course Main Objective(s):					
The objective of this course is to give a detailed and deep knowledge in abstract measure theory and integration over general measure spaces. This course serves as a foundation for current approaches in many areas of mathematics, such as functional analysis, harmonic analysis, partial differential equations, probability theory, fractals, and dynamical systems. The main goal is to develop the basics of abstract measure theory and main properties of the integrals with respect to a measure. The students will be introduced to main theorems in Lebesgue integration and they are expected to understand new mathematical concepts and to use them in practice. They will be able to integrate a measurable function with respect to a measure, to build a measure starting from a countable additive function, and to recognize the main properties of integrable functions.					

2. Teaching mode (mark all that apply)

No	Mode of Instruction	Contact Hours	Percentage
1	Traditional classroom	60	100%
2	E-learning	0	0%
3	Hybrid • Traditional classroom • E-learning	0	0%
4	Distance learning	0	0%

3. Contact Hours (based on the academic semester)

No	Activity	Contact Hours
1.	Lectures	60
2.	Laboratory/Studio	0
3.	Field	0
4.	Tutorial	0
5.	Others (specify)	0
	Total	60

B. Course Learning Outcomes (CLOs), Teaching Strategies and Assessment Methods

Code	Course Learning Outcomes	Code of CLOs aligned with program	Teaching Strategies	Assessment Methods
1.0	Knowledge and understanding			
1.1	To describe the major concepts of measure and integration theory.	K1, K2	4 lecture hours\week	Direct: Regular Exams
1.2	To outline the main results in measure and integration theory with applications.	K1, K2	• 4 lecture hours\week • Self-study	Direct: Short Quizzes
2.0	Skills			
2.1	To develop techniques of proof in Lebesgue integration theory.	S1, S2	Self-study	Direct: • Participations • Short Quizzes
2.2	To develop oral communication and technical writing skills through convergence theorems.	S4	Real-life problems	Direct: Homework and Mini projects
2.3	To use Internet in searching for different kinds of measures	S3	Real-life problems	Direct: Short Quizzes

Code	Course Learning Outcomes	Code of CLOs aligned with program	Teaching Strategies	Assessment Methods
2.4	**To demonstrate out deep proofs in Lp spaces**	**S1, S2**	**Self-study**	**Direct: Participations**
3.0	**Values, autonomy, and responsibility**			
3.1	**To work individually with initiative and responsibility**	**V1, V2**	**Personal questions**	**Direct: Participation**
3.2	**To work in teams with cooperation and positive collaboration**	**V1, V3**	**Teamwork and class discussions.**	**Direct: Homework and Mini projects**

C. Course Content

No	List of Topics	Contact Hours
1.	**Abstract Measure Theory:** Measures and Measurable Sets, Finite Measure, Null Sets, Signed Measure, Invariant Measures. Construction of outer measures. Extension of a set function. Signed measures. The Hahn and Jordan Decompositions.	**15**
2.	**Measurable functions:** Sequence of measurable functions, Properties that hold almost everywhere. Convergence a.e., Convergence in Measure.	**5**
3.	**Integration Theory:** Integration of simple functions, non-negative functions and arbitrarity signed functions. Chebyshev's Inequality.	**10**
4.	**Convergence Theorems:** The Monotone Convergence**,** Fatou's Lemma, Beppo-Levi Theorem**.** The Lebesgue Dominated Convergence Theorem and Application: Parameter-dependent integral and the Absolute Continuity of Integral. Derivatives of Measures, The Radon-Nikodym Theorem, The Vitali-Hahn-Saks Theorem. Modes of convergence. Convergence in mean, Convergence in measure. F. Riesz Theorem.	**10**
5.	**Lp Spaces:** The L1 space. Review of Basic Inequalities, Containment Relations, The L^∞ space. The completeness of Lp for all p. Linear Functionals and Duality, the Riesz Representation Theorem, Density and Approximation Theorems, Separability, Weak Convergence.	**10**
6.	**Product Measures:** Product Sigma-algebra. Product measure. Fubini and Tonelli Theorems and Application, Construction of Lebesgue Measure in R^n	**10**
Total		**60**

D. Students Assessment Activities

No	Assessment Activities *	Assessment timing (in week no)	Percentage of Total Assessment Score
1.	**HomeWorks, Quizzes, Mini projects**	**During the semester**	**30%**
2.	**Midterm**	**Week 9-10**	**30%**
3.	**Final Exam**	**Week 15-16**	**40%**

*Assessment Activities (i.e., Written test, oral test, oral presentation, group project, essay, etc.).

E. Learning Resources and Facilities

1. References and Learning Resources

Essential References	**H. Royden & P. Fitzpatrick:** Real Analysis, Prentice-Hall, 2010 **(Main reference)**
Supportive References	**1. R.F. Bass,** Real Analysis for Graduate Students, version 3.1, 2016 **2. W. Rudin,** Real and Complex Analysis, (Higher Mathematics Series), McGraw Hill, 3rd Edition, 1987. **3. L.F. Richardson,** Measure and Integration, A Concise Introduction to Real Analysis, John Wiley & Sons, 2009.
Electronic Materials	**None**
Other Learning Materials	**None**

2. Educational and Research Facilities and Equipment Required:

Items	Resources
facilities (Classrooms, laboratories, exhibition rooms, simulation rooms, etc.)	▪ **Each class room should be equipped with a whiteboard and a projector.** ▪ **Laboratories should be equipped with computers and an internet connection.**
Technology equipment (projector, smart board, software)	**The rooms should be equipped with data show and Smart Board.**
Other equipment (depending on the nature of the specialty)	**None**

F. Assessment of Course Quality

Assessment Areas/Issues	Assessor	Assessment Methods
Effectiveness of teaching	**Students**	**During the semester and at the end of the course each student will complete two evaluation**

Assessment Areas/Issues	Assessor	Assessment Methods
		forms.
Effectiveness of Students assessment	Instructor	At the end of each semester the course instructor should complete the course report, including a summary of student questionnaire responses appraising progress and identifying changes that need to be made if necessary.
Quality of learning resources	Students	During the semester and at the end of the course each student will complete two evaluation forms.
The extent to which CLOs have been achieved	Instructor	At the end of each semester the course instructor should complete the course report, including a summary of student questionnaire responses appraising progress and identifying changes that need to be made if necessary.
Other	None	

Assessors (Students, Faculty, Program Leaders, Peer Reviewer, Others (specify)

Assessment Methods (Direct, Indirect)

G. Specification Approval

COUNCIL /COMMITTEE	MATHEMATICS AND STATISTICS DEPARTMENT COUNCIL
REFERENCE NO.	8/1446
DATE	05/04/1446 (08/10/2024)