Course Specification

(Postgraduate Programs)

Course Title:	Calculus of Variations
Course Code:	MAT 7239
Program:	Doctor of Philosophy in Mathematics
Department:	Mathematics and Statistics
College:	Science
Institution:	Imam Mohammad Ibn Saud Islamic University
Version:	2024 – V1
Last Revision Date:	None

Table of Contents

A. General information about the course:

1. Course Identification

1. Credit hours:					
4 (4 Lectures, 0 Lab, 0 Tutorial)					
2. Course type					
A.	☐ University	☐ College	☒ Program	☐ Track	☐ Others
B.	☐ Required		☒ Elective		
3. Level/Year at which this course is offered: Level 3 / Year 2					
4. Course general Description:					
This course describes the most important techniques and theoretical results in the calculus of variations: existence and regularity of minimizers and critical point theorems, coercivity, lower semi-continuity, convexity, and weak solutions. As applications, Euler-Lagrange formulation for minimization integral problems will be investigated. The stress will be put on the physical origin of the problems such that minimal surface problems, stationary values of integrals, problems governing the equilibrium states of physical systems.					
5. Pre-requirements for this course (if any):					
None.					
6. Co-requisites for this course (if any):					
None.					
7. Course Main Objective(s):					
This course presents one of the more important methods in nonlinear analysis, namely the variational method. The basic tools and the main theorems will be detailed. A particular attention will be paid to applications: solvability of some boundary value problems for ordinary and partial differential equations as well as some nonlinear integral equations. So the course aims at familiarizing students with the basic techniques used in the calculus of variations emphasizing on variational approaches commonly used to handle some concrete mathematical problems from physics and engineering, nonlinear elasticity, mechanics, ...					

2. Teaching mode (mark all that apply)

No	Mode of Instruction	Contact Hours	Percentage
1	Traditional classroom	60	100%
2	E-learning	0	0%
3	Hybrid • Traditional classroom • E-learning	0	0%
4	Distance learning	0	0%

3. Contact Hours (based on the academic semester)

No	Activity	Contact Hours
1.	**Lectures**	30

2.	Laboratory/Studio	0
3.	Field	0
4.	Tutorial	0
5.	Others (specify)	0
Total		60

B. Course Learning Outcomes (CLOs), Teaching Strategies and Assessment Methods

Code	Course Learning Outcomes	Code of CLOs aligned with program	Teaching Strategies	Assessment Methods
1.0	Knowledge and understanding			
1.1	To outline the main classical and recent techniques in the calculus of variation.	K1, K2	4 lecture hours\week	Direct: Regular Exams
1.2	To state the fundamental notions and concepts commonly used in the calculus of variations.	K1, K2	• 4 lecture hours\week • Self-study	Direct: Short Quizzes
2.0	Skills			
2.1	To develop techniques of proof in calculus of Variations.	S1, S2	Self-study	Direct: • Participations • Short Quizzes
2.2	To develop oral communication and technical writing skills through existence of minimizers.	S4	Real-life problems	Direct: Homework and Mini projects
2.3	To use Internet in searching for different types of convergences.	S3	Real-life problems	Direct: Short Quizzes

Code	Course Learning Outcomes	Code of CLOs aligned with program	Teaching Strategies	Assessment Methods
2.4	To carry out deep proofs in the solvability of some nonlinear equations.	S1, S2	Self-study	Direct: Participations
3.0	Values, autonomy, and responsibility			
3.1	To execute assignments and homework's with independence and responsibility.	V1, V2	Personal questions	Direct: Participation
3.2	To lead work groups.	V1, V3	Teamwork and class discussions.	Direct: Homework and Mini projects

C. Course Content

No	List of Topics	Contact Hours
1.	Differential Calculus in Banach spaces: Gateaux and Frechet Derivatives; Potential Operators, and Integral Representation.	10
2.	Main concepts: Homogeneity, Lower Semi-continuity, Convexity, Coercivity	10
3.	Direct Methods of the Calculus of Variations: Existence of Minimizers for Linear and Nonlinear Functionals; Weak Solutions of Euler-Lagrange Equations in Sobolev Spaces	10
4.	Minimax methods in critical point theory: Ekeland's Variational Principle (Strong and Weak Formulations); Deformation Lemmas, Ambrosetti-Rabinowitz Mountain Pass Theorem.	10
5.	Relaxation and quasi-convexity: Gamma Convergence, Young's Measures, and Singular Perturbation.	10
6.	Solvability of nonlinear equations in Hilbert spaces: Nonlinear	10

	Integral Equations, Semi-Linear Elliptic PDEs, Nonlinear Eigenvalue Problems, and p-Laplacian Problems.	
Total		60

D. Students Assessment Activities

No	Assessment Activities *	Assessment timing (in week no)	Percentage of Total Assessment Score
1.	HomeWorks, Quizzes, Mini projects	During the semester	30%
2.	Midterm	Week 9-10	30%
3.	Final Exam	Week 15-16	40%

*Assessment Activities (i.e., Written test, oral test, oral presentation, group project, essay, etc.).

E. Learning Resources and Facilities

1. References and Learning Resources

Essential References	B. Dacorogna, *Introduction to the Calculus of Variations*; Imperial College Press, London, 2004.
Supportive References	1. D. Motreanu, V.V. Motreanu, and N. Papageorgiou, *Topological and Variational Methods With Applications To Nonlinear Boundary Value Problems*, Springer, 2014. 2. L.C. Evans, *Partial Differential Equations; Graduate Studies In Mathematics*, Providences, RI, 2010.
Electronic Materials	None
Other Learning Materials	None

2. Educational and Research Facilities and Equipment Required:

Items	Resources
facilities (Classrooms, laboratories, exhibition rooms, simulation rooms, etc.)	▪ Each class room should be equipped with a whiteboard and a projector. ▪ Laboratories should be equipped with computers and an internet connection.
Technology equipment (projector, smart board, software)	The rooms should be equipped with data show and Smart Board.

Items	Resources
Other equipment (depending on the nature of the specialty)	None

F. Assessment of Course Quality

Assessment Areas/Issues	Assessor	Assessment Methods
Effectiveness of teaching	Students	During the semester and at the end of the course each student will complete two evaluation forms.
Effectiveness of Students assessment	Instructor	At the end of each semester the course instructor should complete the course report, including a summary of student questionnaire responses appraising progress and identifying changes that need to be made if necessary.
Quality of learning resources	Students	During the semester and at the end of the course each student will complete two evaluation forms.
The extent to which CLOs have been achieved	Instructor	At the end of each semester the course instructor should complete the course report, including a summary of student questionnaire responses appraising progress and identifying changes that need to be made if necessary.
Other	None	

Assessors (Students, Faculty, Program Leaders, Peer Reviewer, Others (specify)

Assessment Methods (Direct, Indirect)

G. Specification Approval

COUNCIL /COMMITTEE	MATHEMATICS AND STATISTICS DEPARTMENT COUNCIL
REFERENCE NO.	8/1446
DATE	05/04/1446 (08/10/2024)