Course Specification

(Postgraduate Programs)

Course Title:	**Approximation Theory**
Course Code:	**MAT 7247**
Program:	**Doctor of Philosophy in Mathematics**
Department:	**Mathematics and Statistics**
College:	**Science**
Institution:	**Imam Mohammad Ibn Saud Islamic University**
Version:	**2024 – V1**
Last Revision Date:	**None**

Table of Contents

A. General information about the course:

1. Course Identification

1. Credit hours:					
4 (4 Lectures, 0 Lab, 0 Tutorial)					
2. Course type					
A.	☐ University	☐ College	☒ Program	☐ Track	☐ Others
B.	☐ Required		☒ Elective		
3. Level/Year at which this course is offered: Level 3 / Year 2					
4. Course general Description:					
Approximation Theory is concerned with approximating functions into more simple building blocks, such as polynomials, Splines, Wavelets, or other special functions. The primary focus is analyzing how properties, such as smoothness or variation, of the function govern the rates of convergence of the approximating classes of functions. The efficient solutions of such problems are of great importance for computing, and this module will introduce the mathematical theory behind many approximation methods in common use.					
5. Pre-requirements for this course (if any):					
None.					
6. Co-requisites for this course (if any):					
None.					
7. Course Main Objective(s):					
The course has been designed to emphasize the theory of approximation theory which serves either directly as the basis for much of numerical analysis or at the very least enters in a critical way in its development. The primary goal of the discipline is to approximate functions by simpler functions with interesting properties, e.g. by polynomials, splines, etc. This course also covers the necessary background for study in modern topics in pure and applied mathematics, which include Lagrange and Hermite Interpolation, Approximation of Linear Equations, Polynomials Approximation, Least Squares Approximation, Rational Approximation, Cubic and B-Splines.					

2. Teaching mode (mark all that apply)

No	Mode of Instruction	Contact Hours	Percentage
1	Traditional classroom	60	100%
2	E-learning	0	0%
3	Hybrid • Traditional classroom • E-learning	0	0%
4	Distance learning	0	0%

3. Contact Hours (based on the academic semester)

No	Activity	Contact Hours
1.	Lectures	60
2.	Laboratory/Studio	0
3.	Field	0
4.	Tutorial	0
5.	Others (specify)	0
Total		60

B. Course Learning Outcomes (CLOs), Teaching Strategies and Assessment Methods

Code	Course Learning Outcomes	Code of CLOs aligned with program	Teaching Strategies	Assessment Methods
1.0	Knowledge and understanding			
1.1	To interpret some theories and concepts used in approximation theory.	K1, K2	4 lecture hours\week	Direct: Regular Exams
1.2	To describe the fundamental methods and theoretical basis of approximation theory.	K1, K2	• 4 lecture hours\week • Self-study	Direct: Short Quizzes
2.0	Skills			
2.1	To develop techniques of proof in interpolation and approximations.	S1, S2	Self-study	Direct: • Participations • Short Quizzes
2.2	To develop oral communication and technical writing skills through error analysis and best approximations.	S4	Real-life problems	Direct: Homework and Mini projects
2.3	To use Internet in searching for some algorithms.	S3	Real-life problems	Direct: Short Quizzes
2.4	To carry out deep proofs when using B-splines	S1, S2	Self-study	Direct: Participations
3.0	Values, autonomy, and responsibility			
3.1	To work independently.	V1, V2	Personal questions	Direct: Participation
3.2	To work with collaboration and cooperation.	V1, V3	Teamwork and class discussions.	Direct: Homework and Mini projects

C. Course Content

No	List of Topics	Contact Hours
1.	**Polynomial interpolation:** The Lagrange Interpolation. The error in Polynomial Interpolation. The norm of the Lagrange Interpolation Operator. Hermite Interpolation.	8
2.	**The uniqueness of best approximations:** Convexity Conditions. Conditions For the Uniqueness of Best Approximation. The Continuity of Approximation Operators. The L1, L2, L ∞ - Norms.	7
3.	**Polynomial interpolation:** The Lagrange Interpolation. The error in Polynomial Interpolation. The norm of the Lagrange Interpolation Operator. Hermite Interpolation.	7
4.	**Approximate Solution of Over-Determined System of Linear Equations:** Chebyshev Solution of Linear Equations. Systems of Equations with one Unknown. Characterization of the Solution. The Special Cases, Polya's Algorithm. The Descent Algorithm. Convex Programming.	7
5.	**Least-Squares Approximation and related topics:** Inner Products. Various Orthogonal Systems. Orthogonal Systems of Polynomials. Convergence of Orthogonal Expansion. Approximation by Series of Chebyshev Polynomials. Discrete Least Squares Approximation. Jackson Theorems.	7
6.	**Rational Approximation:** Conversion of rational functions to continued fractions. Existence of best rational approximation. Characterization of best approximation. Uniqueness; Continuity of best approximation operators Algorithms. Padè approximation and its generalizations. Continued fractions.	7
7.	**Interpolation by pricewise polynomials:** Local Interpolation methods. Cubic spline interpolation. End condition for Cubic Splines Interpolation. Interpolating Splines of Other Degrees.	7
8.	**B-Splines:** The Parameters of a Spline Function. The Form of B-Splines. B-Splines as basic Functions. A Recurrence Relation for B-Splines. The Schoenberg-Whitney Theorem. Convergence Properties of Spline Approximation.	10
	Total	**60**

D. Students Assessment Activities

No	Assessment Activities *	Assessment timing (in week no)	Percentage of Total Assessment Score
1.	HomeWorks, Quizzes, Mini projects	During the semester	30%
2.	Midterm	Week 9-10	30%
3.	Final Exam	Week 15-16	40%

*Assessment Activities (i.e., Written test, oral test, oral presentation, group project, essay, etc.).

E. Learning Resources and Facilities

1. References and Learning Resources

Essential References	**N.I. Achieser,** *Theory of Approximations*, Dover, NY, 1992 (2004).
Supportive References	• **M.J.D. Powell,** *Approximation Theory and Methods*, Cambridge 1981. • **E. W. Cheney,** *An Introduction to Approximation Theory*, McGraw-Hill, 1966.
Electronic Materials	**None**
Other Learning Materials	**None**

2. Educational and Research Facilities and Equipment Required:

Items	Resources
facilities (Classrooms, laboratories, exhibition rooms, simulation rooms, etc.)	▪ **Each class room should be equipped with a whiteboard and a projector.** ▪ **Laboratories should be equipped with computers and an internet connection.**
Technology equipment (projector, smart board, software)	**The rooms should be equipped with data show and Smart Board.**
Other equipment (depending on the nature of the specialty)	**None**

F. Assessment of Course Quality

Assessment Areas/Issues	Assessor	Assessment Methods
Effectiveness of teaching	**Students**	**During the semester and at the end of the course each student will complete two evaluation forms.**
Effectiveness of Students assessment	**Instructor**	**At the end of each semester the course instructor should complete the course report, including a summary of student questionnaire responses appraising progress and identifying changes that need to be made if necessary.**
Quality of learning resources	**Students**	**During the semester and at the end of the course each student will complete two evaluation forms.**
The extent to which CLOs have been achieved	**Instructor**	**At the end of each semester the course instructor should complete the course report,**

Assessment Areas/Issues	Assessor	Assessment Methods
		including a summary of student questionnaire responses appraising progress and identifying changes that need to be made if necessary.
Other	None	

Assessors (Students, Faculty, Program Leaders, Peer Reviewer, Others (specify)

Assessment Methods (Direct, Indirect)

G. Specification Approval

COUNCIL /COMMITTEE	MATHEMATICS AND STATISTICS DEPARTMENT COUNCIL
REFERENCE NO.	8/1446
DATE	05/04/1446 (08/10/2024)