Kingdom of Saudi Arabia

Imam Mohammed Ibn Saud

Islamic University

College of Science – Physics

Department



المملكة العربية السعودية جامعة الإمام محمد بن سعود الإسلامية كلية العلوم - قسم الفيزياء

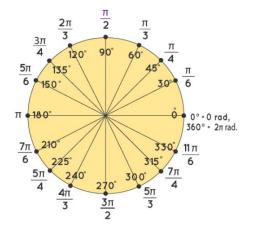
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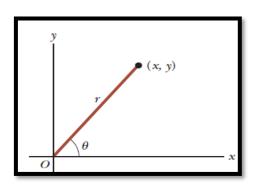
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Chapter1_Coordinate Systems

1.1 Polar and Cartesian coordinate system

- r is the distance from the origin O to the point having Cartesian coordinates (x, y).
- θ is the angle between a fixed axis and a line drawn from the origin to the point.
- The fixed axis is often the positive *x-axis*, and θ is usually measured counterclockwise from it.
- The polar coordinates are (r, θ) . Note: θ is often expressed in radians.





Formulas:

Cartesian coordinates in terms of polar coordinates
$$x = r\cos\theta$$
$$y = r\sin\theta$$
Polar coordinates in terms of Cartesian coordinates
$$\tan\theta = \frac{y}{x}$$
$$r = \sqrt{x^2 + y^2}$$

Distance between two point (d)

$$d = \sqrt{(\Delta x)^2 + (\Delta y)^2}$$

 $\Delta x = x_2 - x_1$ and $\Delta y = y_2 - y_1$

Example.1

Convert the following (given in polar coordinates) (2, 45°) to Cartesian coordinates

Solution:

we have r = 2, $\theta = 45^{\circ}$

$$x = r \cos\theta = 2 \cos(45^\circ) = 2 (\sqrt{1/2}) = \sqrt{2}$$

$$y = r \sin\theta = 2 \sin(45^\circ) = 2 (\sqrt{1/2}) = \sqrt{2}$$

Cartesian coordinates are $(\sqrt{2}, \sqrt{2})$

Example 2

Calculate the polar coordinates given the Cartesian coordinates (2, 2)?

Solution:

For (2, 2), we have x = 2, and y = 2.

Therefore,
$$r^2 = x^2 + y^2 = 4 + 4 = 8$$
, and $r = \sqrt{2}$

We have $tan \theta = y/x$, so

$$\theta = tan^{-1} y/x = tan^{-1} 2/2.$$

$$\theta = 45^{\circ}$$

Since this point is in the first quadrant, we have $\theta = \pi/4$. (or 45°) Therefore the polar coordinates for the point (2, 2) are $(\sqrt{2}, \pi/4)$.

1.	Convert each of the following (x,y) points into polar coordinates?
a. ((1, 1)
b.	(-3, 4)
c. ((1,3)
••••	
d. ((5, -5)

2. (Convert each of the following points into Cartesian coordinates.
	A. $(3, \pi/3)$
	b. $(2, 3\pi/2)$
	c. $(6, -5\pi/6)$
3.	The polar coordinates of a point are $r = 5.50$ m and $\theta = 240^{\circ}$. What are the
Ca	rtesian coordinates of this point?
•••••	
••••	
••••	
••••	

4. Two points in a plane have polar coordinates (2.50 m, 30.0°) and (3.80 m,		
120.0°). Determine (a) the Cartesian coordinates of these points and (b) the		
distance between them.		
4 A \$7 4		
1.2 Vectors.		
1.2.1 Physical Quantities		
• A <u>scalar quantity</u> is completely specified by a single value (magnitude)		
with an appropriate unit and has no direction.		
Examples: Distance, mass, speed and time.		
■ A <u>vector quantity</u> is completely specified by a number with an appropriate		
unit (the magnitude of the vector) plus a direction.		
Examples: velocity, displacement, acceleration and force.		
Practice Questions		
1. The scalar quantity has:		
a) both magnitude and direction. b) magnitude only. c) direction only.		
2. Which of the fo.llowing is a scalar quantity:		
a) Speed b) Velocity c) Displacement d) None of these		

3. Which of the following is a vector quantity?

- a) Mass
- b) Density
- c) Temperature
- d) Force

4. The vector quantity has:

a) both magnitude and direction. b) magnitude only. c) direction only

5. Which of the following is a scalar quantity:

- a) Distance
- b) Velocity
- c) Displacement
- d) None of these

1.2.2 Components of a Vector and Unit Vectors:

Formulas:

$$\vec{\mathbf{A}} = \vec{\mathbf{A}}_x + \vec{\mathbf{A}}_y$$

•The component A_x represents the projection of \vec{A} along the x axis, and the component A_y represents the projection of \vec{A} along the y axis.

•These components can be positive or negative.

•The components of \overrightarrow{A} are

$$A_x = A \cos \theta$$

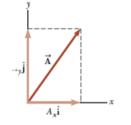
$$A_{y} = A \sin \theta$$

 $\vec{\mathbf{A}} = A_{\mathbf{x}}\hat{\mathbf{i}} + A_{\mathbf{y}}\hat{\mathbf{j}}$

•The magnitude and direction of \overrightarrow{A}

$$A = \sqrt{A_x^2 + A_y^2}$$

$$\theta = \tan^{-1} \left(\frac{A_{y}}{A_{x}} \right)$$



Example.1

1. Find the x and y components of vector A having a magnitude of 12 m and making an angle of 60° degrees with the positive x-axis.

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Solution:

The magnitude of vector A = 12 m, and it makes an angle $\theta = 60^{\circ}$.

The x component of the vector = $A_x = A\cos\theta = 12.\cos 60^\circ = 6 \text{ m}$

The y component of the vector = $A_y = Asin\theta = 12.sin 60^{\circ} = 10.4 \text{ m}$

Exmaple.2

Find the magnitude and counter-clockwise angle (direction) it makes with the x-axis axis of the vector $\vec{A} = (8\hat{\imath} + 6\hat{\jmath}) m$

Solution:

$$A = \sqrt{A_x^2 + A_y^2}$$

$$A = \sqrt{8^2 + 6^2} = 10 m$$

$$\theta = \tan^{-1} \left(\frac{A_{y}}{A_{x}} \right)$$

$$\theta = tan^{-1}(\frac{6}{8}) = 36.87^{\circ}$$

1. Find the magnitude and direction (with respect to the positive x-axis) of the
$\operatorname{vector} \vec{A} = (3\hat{\imath} + 4\hat{\jmath}) \ m.$
2. Vector A has components vector $A_x = -8$ m and $A_y = 10$ m. Find the magnitude
of the vector and the counter-clockwise angle (direction) it makes with the
positive x-axis.

If the magnitude and counter clockwise direction (with respect to the we x-axis) of $\vec{B} = (-6\hat{\imath} + 4\hat{\jmath}) m$
the magnitude and direction with positive <i>x</i> -axis of vector $\overrightarrow{B} = 8\hat{\jmath}$) <i>m</i>
If the components of vector of \vec{A} a which has a magnitude of 5 and making the of 30° with positive x-axis.
ele of 30° with positive x-axis.

1.2.3 Adding Vectors

Formulas:

The resultant vector $\overrightarrow{R} = \overrightarrow{A} + \overrightarrow{B}$ is

$$\vec{\mathbf{R}} = (A_x \hat{\mathbf{i}} + A_y \hat{\mathbf{j}}) + (B_x \hat{\mathbf{i}} + B_y \hat{\mathbf{j}})$$

or

$$\vec{\mathbf{R}} = (A_x + B_x)\hat{\mathbf{i}} + (A_y + B_y)\hat{\mathbf{j}}$$

Because $\vec{\mathbf{R}} = R_x \hat{\mathbf{i}} + R_y \hat{\mathbf{j}}$, we see that the components of the resultant vector are

$$R_x = A_x + B_x$$

$$R_{y} = A_{y} + B_{y}$$

Example.1

Consider the two vectors $\mathbf{A} = 3\ddot{\mathbf{i}} - 2\ddot{\mathbf{j}}$ and $\mathbf{B} = -\ddot{\mathbf{i}} - 4\ddot{\mathbf{j}}$. Calculate (a) $\mathbf{A} + \mathbf{B}$,

(b) $\mathbf{A} - \mathbf{B}$, (c) $|\mathbf{A} + \mathbf{B}|$, (d) $|\mathbf{A} - \mathbf{B}|$, and (e) the directions of $\mathbf{A} + \mathbf{B}$ and $\mathbf{A} - \mathbf{B}$.

Solution:

(a)
$$(\mathbf{A} + \mathbf{B}) = (3\hat{\mathbf{i}} - 2\hat{\mathbf{j}}) + (-\hat{\mathbf{i}} - 4\hat{\mathbf{j}}) = 2\hat{\mathbf{i}} - 6\hat{\mathbf{j}}$$

(b)
$$(\mathbf{A} - \mathbf{B}) = (3\hat{\mathbf{i}} - 2\hat{\mathbf{j}}) - (-\hat{\mathbf{i}} - 4\hat{\mathbf{j}}) = \boxed{4\hat{\mathbf{i}} + 2\hat{\mathbf{j}}}$$

(c)
$$|\mathbf{A} + \mathbf{B}| = \sqrt{2^2 + 6^2} = \boxed{6.32}$$

(d)
$$|\mathbf{A} - \mathbf{B}| = \sqrt{4^2 + 2^2} = \boxed{4.47}$$

(e)
$$\theta_{|\mathbf{A}+\mathbf{B}|} = \tan^{-1}\left(-\frac{6}{2}\right) = -71.6^{\circ} = \boxed{288^{\circ}}$$
$$\theta_{|\mathbf{A}-\mathbf{B}|} = \tan^{-1}\left(\frac{2}{4}\right) = \boxed{26.6^{\circ}}$$

Practice Questions

1.

Consider two vectors: $\vec{\mathbf{A}} = 3\hat{\mathbf{i}} - \hat{\mathbf{j}}$ and $\vec{\mathbf{B}} = -2\hat{\mathbf{i}} - 4\hat{\mathbf{j}}$ calculate $\vec{\mathbf{A}} + 2\vec{\mathbf{B}}$, $\vec{\mathbf{A}} - \vec{\mathbf{B}}$,
magnitude of $\overrightarrow{A} - \overrightarrow{B}$, and magnitude of $\overrightarrow{A} - \overrightarrow{B}$.
2. Let vector $\vec{A} = (3\hat{\imath} + 3\hat{\jmath})\frac{m}{s}$ and vector $\vec{B} = (-\hat{\imath} + 5\hat{\jmath})\frac{m}{s}$. Use the formulas
above to find $\vec{D} = \vec{A} + \vec{B}$ and calculate the magnitude and direction of the
vector \overrightarrow{D} .

1.2.4 The Scalar Product of Two Vectors:

The result of the product is scalar.

Formulas:

The scalar products of these unit vectors are

$$\hat{\mathbf{i}} \cdot \hat{\mathbf{i}} = \hat{\mathbf{j}} \cdot \hat{\mathbf{j}} = \hat{\mathbf{k}} \cdot \hat{\mathbf{k}} = 1$$

$$\hat{\mathbf{i}} \cdot \hat{\mathbf{j}} = \hat{\mathbf{i}} \cdot \hat{\mathbf{k}} = \hat{\mathbf{j}} \cdot \hat{\mathbf{k}} = 0$$

Two vectors **A** and **B** can be expressed in unit vector form as

$$\vec{\mathbf{A}} = A_x \hat{\mathbf{i}} + A_y \hat{\mathbf{j}} + A_z \hat{\mathbf{k}}$$

$$\overrightarrow{\mathbf{B}} = B_x \hat{\mathbf{i}} + B_y \hat{\mathbf{j}} + B_z \hat{\mathbf{k}}$$

$$\overrightarrow{\mathbf{A}} \cdot \overrightarrow{\mathbf{B}} = A_x B_x + A_y B_y + A_z B_z$$

$$\vec{A} \cdot \vec{B} = |\vec{A}| |\vec{B}| \cos \theta$$

Example.1

If $\vec{A} = -2\mathbf{i} + 3\mathbf{j} + 5\mathbf{k}$ and $\vec{B} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$ are two vectors, then find the value of the dot product of these two vectors.

Solution:

$$\overrightarrow{A.B} = (-2)(1) + (3)(2) + (5)(3)$$

= -2 + 6 + 15 = 19

Practice Questions

1.If $\vec{A} = -4\mathbf{i} + 5\mathbf{j} + 2\mathbf{k}$ and $\vec{B} = \mathbf{i} + 2\mathbf{j}$ are two vectors, then find the value of the dot product of these two vectors.

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2. If $\vec{C} = \mathbf{i} + \mathbf{k}$ and $\vec{D} = \mathbf{i} + 4\mathbf{j} + 6\mathbf{k}$ are two vectors, then find the value of the dot product of these two vectors.

3. What is the dot product of two vectors of magnitude 3 and 5, if the angle
between them is 60°?
ightarrow
4. If $\vec{A} = 2\mathbf{i} - 5\mathbf{j} - 2\mathbf{k}$ and $\vec{B} = \mathbf{i} + 3\mathbf{k}$ are two vectors, then find the value of the
dot product of these two vectors.

1.2.4 Vector (cross) Product

The result of the product is a <u>vector</u>.

Formulas:

$$\hat{\mathbf{i}} \times \hat{\mathbf{i}} = \hat{\mathbf{j}} \times \hat{\mathbf{j}} = \hat{\mathbf{k}} \times \hat{\mathbf{k}} = 0$$

$$\hat{\mathbf{i}} \times \hat{\mathbf{j}} = -\hat{\mathbf{j}} \times \hat{\mathbf{i}} = \hat{\mathbf{k}}$$

$$\hat{\mathbf{j}} \times \hat{\mathbf{k}} = -\hat{\mathbf{k}} \times \hat{\mathbf{j}} = \hat{\mathbf{i}}$$

$$\hat{\mathbf{k}} \times \hat{\mathbf{i}} = -\hat{\mathbf{i}} \times \hat{\mathbf{k}} = \hat{\mathbf{j}}$$

$$\vec{A} \times \vec{B} = |\vec{A}| |\vec{B}| \sin \theta$$

$$\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \mathbf{\hat{i}} & \mathbf{\hat{j}} & \mathbf{\hat{k}} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \begin{vmatrix} A_y & A_z \\ B_y & B_z \end{vmatrix} \mathbf{\hat{i}} - \begin{vmatrix} A_x & A_z \\ B_x & B_z \end{vmatrix} \mathbf{\hat{j}} + \begin{vmatrix} A_x & A_y \\ B_x & B_y \end{vmatrix} \mathbf{\hat{k}}$$

$$\mathbf{A} \times \mathbf{B} = (A_y B_z - A_z B_y) \,\hat{\mathbf{i}} - (A_x B_z - A_z B_x) \,\hat{\mathbf{j}} + (A_x B_y - A_y B_x) \hat{\mathbf{k}}$$

Example.1

For the two vector $\overrightarrow{A} = (2\hat{\imath} + 3\hat{\jmath}) \, m/s$ and vector $\overrightarrow{B} = (-\hat{\imath} + 2\hat{\jmath}) \, m/s$, Find $\overrightarrow{A} \times \overrightarrow{B}$ and verify that $\overrightarrow{A} \times \overrightarrow{B} = -\overrightarrow{B} \times \overrightarrow{A}$.

Solution:

Write the cross product of the two vectors: $\vec{A} \times \vec{B} = (2\hat{i} + 3\hat{j}) \times (-\hat{i} + 2\hat{j})$

Perform the multiplication: $\vec{\mathbf{A}} \times \vec{\mathbf{B}} = 2\hat{\mathbf{i}} \times (-\hat{\mathbf{i}}) + 2\hat{\mathbf{i}} \times 2\hat{\mathbf{j}} + 3\hat{\mathbf{j}} \times (-\hat{\mathbf{i}}) + 3\hat{\mathbf{j}} \times 2\hat{\mathbf{j}}$

 $\vec{\mathbf{A}} \times \vec{\mathbf{B}} = 0 + 4\hat{\mathbf{k}} + 3\hat{\mathbf{k}} + 0 = 7\hat{\mathbf{k}}$

To verify that $\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$, evaluate $\vec{B} \times \vec{A} = (-\hat{i} + 2\hat{j}) \times (2\hat{i} + 3\hat{j})$

 $\vec{\mathbf{B}} \times \vec{\mathbf{A}}$:

Perform the multiplication:

 $\vec{\mathbf{B}} \times \vec{\mathbf{A}} = (-\hat{\mathbf{i}}) \times 2\hat{\mathbf{i}} + (-\hat{\mathbf{i}}) \times 3\hat{\mathbf{j}} + 2\hat{\mathbf{j}} \times 2\hat{\mathbf{i}} + 2\hat{\mathbf{j}} \times 3\hat{\mathbf{j}}$

 $\vec{\mathbf{B}} \times \vec{\mathbf{A}} = 0 - 3\hat{\mathbf{k}} - 4\hat{\mathbf{k}} + 0 = -7\hat{\mathbf{k}}$

Therefore, $\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$.

Example.2

Two vectors are given by $\mathbf{A} = -3\hat{\mathbf{i}} + 4\hat{\mathbf{j}}$ and $\mathbf{B} = 2\hat{\mathbf{i}} + 3\hat{\mathbf{j}}$.

Find (a) $\mathbf{A} \times \mathbf{B}$ and (b) the angle between \mathbf{A} and \mathbf{B} .

Solution:

(a)
$$\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ -3 & 4 & 0 \\ 2 & 3 & 0 \end{vmatrix} = \boxed{-17.0\hat{\mathbf{k}}}$$

(b)
$$|\mathbf{A} \times \mathbf{B}| = |\mathbf{A}| |\mathbf{B}| \sin \theta$$
$$17 = 5\sqrt{13} \sin \theta$$
$$\theta = \arcsin\left(\frac{17}{5\sqrt{13}}\right) = \boxed{70.6^{\circ}}$$

1. If the vector $\overrightarrow{A} = (-6\hat{\imath} + 10\hat{\jmath}) m/s$ and vector $\overrightarrow{C} = (12\hat{\imath} - 20\hat{\jmath}) m/s$,
Find $\overrightarrow{A} \times \overrightarrow{C}$.
→ (TA TA) → (10A 00A) /
2. If the vector $\vec{a} = (5\hat{\imath} + 5\hat{\jmath}) m/s$ and vector $\vec{b} = (12\hat{\imath} - 20\hat{\jmath}) m/s$,
Find $\vec{a} \times \vec{b}$. And the angle between them.

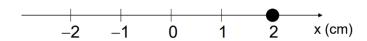
	s the magnitude of the cross product of two vectors of magnitude 10
and 6 N, 1	f the angle between them is 30°.
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•••••	
4. If the v	ector $\vec{a} = (3\hat{\imath} + 2\hat{\jmath} + 4\hat{k}) m/s$ and vector $\vec{b} = (2\hat{\imath} - 5\hat{\jmath} + 2\hat{k}) m/s$
Find \vec{a} x \vec{b}	and the angle between them.
5.If the ve	ector $\vec{a} = (6\hat{\imath} + 2\hat{\jmath} - 2\hat{k}) m/s$ and vector $\vec{b} = (\hat{\imath} - 7\hat{\jmath} + 2\hat{k}) m/s$,
Find \vec{a} x \vec{b}	and the angle between them.

Chapter 2 Motion

2.1 Motion in one Dimension

The position (x) of an object describes its location relative to some origin or other reference point.

Example



Solution:

The position of the ball above is x = +2 cm.

2.1.1 Displacement (Δx) and Distance

$$\Delta x = x_f - x_i$$

Distance is the length of a path followed by a particle.

Example.1

A ball is initially at x = +2 cm and is moved to x = -2 cm. What is the displacement of the ball?

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Solution:

$$\Delta x = -2 \,\mathrm{cm} - 2 \,\mathrm{cm} = -4 \,\mathrm{cm}.$$

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1. At 3 PM a car is located 20 km south of its starting point. One hour later it is
96 km farther south. After two more hours it is 12 km south of the original starting
point. What is the displacement of the car between 3 PM and 6 PM?
2. A car travelled a long straight road 100 km east then 50 km west. Calculate
the distance and displacement of the car?

2.1.2 Average velocity and average speed:

Average Velocity is the change in position Δx divided by the time interval Δt .

Formulas:

Average velocity	Average speed
$V_{ m x,avg}$	V_{avg}
It is a vector quantity	It is a scalar quantity
It's unit is meter per second in SI Unit m/s	It's unit is meter per second in SI Unit m/s
It is a change of displacement dividing by a time interval	It is a total of distance dividing by a total time
$V_{x,avg} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$ OR	$V_{avg} = \frac{total\ distance}{total\ time}$
$V_{x,avg} = \frac{v_{x1} + v_{x2}}{2}$	$V_{\text{avg}} = \frac{d_1 + d_2 + \cdots}{t_1 + t_2 + \cdots}$
$V_{x,avg} \text{ it can be} \begin{cases} (+) \text{ at } \Delta x > 0 \\ (-) \text{ at } \Delta x < 0 \\ (0) \text{at } \Delta x = 0 \end{cases}$	

Example.1

A car travels 60 km in the first hour and 80 km in the second hour. Calculate its average speed.

Solution:

$$v_{av} = \frac{\text{total distance}}{\text{total time}}$$
$$= (60+80)/(1+1) = 140/2 = 70 \text{ km/hr}$$

Example.2

A truck moves in a straight line down the track. At a time 1s after the start of the movement, the car is at $x_1 = 19$ m to the right of the origin. Then, at 4s after the start, it is at $x_2 = 277$ m from the origin. Find the average velocity for the truck.

Solution:

$$v_{av-x} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t}$$
 $v_{av-x} = \frac{277 - 19}{4 - 1} = 86m/s$

1. To complete a journey of 200 km, a truck requires 4 hrs. Calculate the
average speed of the truck.
2. Between points A and B, the average speed of a car is 30 m/s, and between B
and C is 20 m/s. Calculate the average speed between A and C, if the time taken
in the stated sections is 20s and 10s, respectively.
3. Find the average velocity of an object when the change in displacement is 81m,
and the total time taken is 9 s.

2.1.3 Instantaneous velocity (v_x) and speed (S)

Formulas:

$$v_x = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$$

$$v_x = \frac{dx}{dt}$$

The **speed** of a particle is the magnitude of its instantaneous velocity

$$s = |v_{av}|$$

Example.1

If the position of a particle varies with respect to time and is given as $(x=6t^2+2t+4)$ m, find the instantaneous velocity and speed at t=1second.

Solution:

$$v_x = \frac{dx}{dt}$$

$$v_x = \frac{d(6t^2 + 2t + 4)}{dt} = 12t + 2$$

at
$$t = 1s$$
, $v_x = 12(1) + 2 = 14$ m/s

1. The position function of an object is $x(t) = t^2 - 3t$. Compute the instantaneous
velocity of the object at $t = 7$ seconds.

•••	
2	The position function of an object is given by $x(t) = 4t^3 + 3t + 1$. Compute to
ir	stantaneous velocity and speed of the object at $t = 4$ seconds.
•••	
•••	
•••	

Formulas:

Average Acceleration

$$a_{av-x} = \frac{v_{2x} - v_{1x}}{t_2 - t_1} = \frac{\Delta v_x}{\Delta t}$$

Unit m/s^2

Instantaneous Acceleration

$$a_x = \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} = \frac{dv_x}{dt} = \frac{d^2 x}{dt^2}$$

$$a_x = \frac{dv}{dt}$$

Unit m/s²

Example.1

A racehorse coming out of the gate accelerates from rest to a velocity of 15.0 m/s in 2.0 s. What is its average acceleration?

Solution: from rest v_{Ix} =zero

$$a_{av-x} = \frac{v_{2x} - v_{1x}}{t_2 - t_1} = \frac{\Delta v_x}{\Delta t}$$

$$a_{av} = \frac{15-0}{2-0} = 7.5 \text{ m/s}^2$$

Example.2

The position of a particle on a line is given by $x(t) = t^3 - 3t^2 - 6t + 5$, where t is measured in seconds and x is measured in meter. Find the acceleration of the particle at the end of 2 seconds.

Solution:

$$a_x = \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} = \frac{dv_x}{dt} = \frac{d^2 x}{dt^2}$$

$$a_x = 6t - 6 = (6x2) - 6 = 6 \text{ m/s}^2$$

1. The speed of a traveling bus increases from 15 m/s to 20 m/s in 4 seconds.	
Find the average acceleration of the bus.	
	••
	•••

2. A new car can accelerate from 0 to 60 m/s in just 7 seconds. Calculate the average acceleration of the car.	
	••••
	••••
1. The position of a particle on a line is given by $x(t) = t^4 + 3 t^2 + 12 t$, where	
measured in seconds and x is measured in meters. Find the acceleration of	ιne
particle at the end of 1 second.	
	••••
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2. If the position of an object is given as $x(t) = 0.5 t^3 + 2t$. Calculate the	
nstantaneous acceleration after 3 seconds of motion?	
	• • •
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Example.3	
If the velocity of the object is given as $v = 10 t^2 + 2t$. Calculate the instantaneo	us
acceleration after 2 seconds of motion?	
Solution:	

 $a_x = \frac{dv}{dt} = 20t + 2 = 20(2) + 2 = 40 + 2 = 42m / s^2$

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Practice (Juestions

1. A particle is in motion and is accelerating. The functional form of its velocity
is $v(t)=20t-5t^2 m/s$. Find the instantaneous acceleration at $t=1,3$, and 5 s.
2. The functional form of velocity of a moving object is $v(t)=6t^2-10t$ m/s. Find
the instantaneous acceleration at $t = 1.5$ s.
3. The position of a particle moving on the x axis is given by $x(t) = 20 + 4 t^2$
$3t^3$. (a) What is its instantaneous velocity at $t = 2$ seconds? (b) Find the average
velocity in the interval between 1 sec and 6 sec.

2.1.4 Kinematic Equations of Motion with Constant Acceleration in One Dimension and Free Falling

Formulas

x-direction

$$v = v_0 + at$$

$$x = x_0 + v_0 t + \frac{1}{2} a t^2$$

$$v^2 = v_0^2 + 2a(x - x_0)$$

$$x = x_0 + \frac{1}{2} (v_0 + v) t$$

$$\Delta x = v_0 t + rac{1}{2} a t^2$$

particle has position x_0 and velocity v_0 at time t = 0; it has position x and velocity v at time t.

Free Fall Kinetematic Equations

Because a = -g for free fall and we use y instead of x, we have the following



acceleration due to gravity (g) $g = 9.80 \text{ m/s}^2$ (downward)

Example.1 (*x*-direction)

A car at rest waiting to merge onto a highway. It accelerates at 4 m/s^2 for 7 s. What is the car's final velocity?

Solution:

$$v = v_0 + at$$

 $v = 0 + (4 \cdot 7)$
 $= 28 \text{ m/s}$
 $v = 28 \text{ m/s}$

Example.2

A ball rolls toward a hill at 3 m/s. It rolls down the hill for 5 s and has a final velocity of 18 m/s. What was the ball's acceleration as it rolled down the hill? **Solution:**

$$v-v_0=at$$
 $a=rac{v-v_0}{t}$ $a=rac{18 ext{ m/s}-3 ext{ m/s}}{5 ext{ s}}$ $a=3 ext{ m/s}^2$

Example.3

A rocket is cruising through space with a velocity of 50 m/s and burns some fuel to create a constant acceleration of 10 m/s². How far will it have travelled after 5 s?

$$\Delta x = v_0 t + rac{1}{2} a t^2$$
 $\Delta x = (50 \mathrm{\ m/s})(5 \mathrm{\ s}) + rac{1}{2} (10 \mathrm{\ m/s}^2)(5 \mathrm{\ s})^2$ $\Delta x = 250 \mathrm{\ m} + rac{1}{2} (10 \mathrm{\ m/s}^2)(25 \mathrm{\ s}^2)$ $\Delta x = 250 \mathrm{\ m} + rac{1}{2} (250 \mathrm{\ m})$ $\Delta x = 250 \mathrm{\ m} + 125 \mathrm{\ m}$ $\Delta x = 375 \mathrm{\ m}$

Example.4

A car exiting the highway begins with a speed of 25 m/s and travels down a 100 m-long exit ramp with a deceleration (negative acceleration) of 3 m/s². What is the car's velocity at the end of the exit ramp?

Solution:

$$v^2 = (25 \text{ m/s})^2 + 2 \cdot (-3 \text{ m/s}^2) \cdot 100 \text{ m}$$
 $v^2 = 625 \text{ m}^2/\text{ s}^2 + 2 \cdot (-3 \text{ m/s}^2) \cdot 100 \text{ m}$
 $v^2 = 625 \text{ m}^2/\text{ s}^2 + 2 \cdot (-300 \text{ m}^2/\text{ s}^2)$
 $v^2 = 625 \text{ m}^2/\text{ s}^2 - 600 \text{ m}^2/\text{ s}^2$
 $v^2 = 25 \text{ m}^2/\text{ s}^2$
 $v = 5 \text{ m/s}$

1. Calculate the distance travelled by a bus with an initial velocity of 30 m/s and
that increased its speed to 60 m/s in 20 seconds, assuming acceleration is constant.
2. A car initially traveling along a straight line from rest accelerates with a
constant acceleration of 6 m/s ² for 5 seconds. Calculate the final velocity.

. A body travelling with a velocity of 120 ms ⁻¹ accelerates uniformly at a of 20 ms ⁻² for a period of 40 s. Calculate the velocity and the distance traven 40 s.	
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of 20 ms ⁻² for a period of 40 s. Calculate the velocity and the distance traven 40 s.	
n 40 s.	
	velled
	•••••
6. A car initially traveling along a straight highway at 13 m/s accelerates venstant acceleration of 3 m/s ² .	with a
) What is the velocity of the car after 5s?	
) what distance does the car travel during its 2s of acceleration?	
	•••••



2.1.5 Free falling (y-direction)

Example.1

An object is dropped from rest from the top of a tall building. It hits the ground 5s after it is dropped. What is the height of the building?

Solution:

$$\Delta y = v_i - \frac{1}{2}gt^2$$

$$\Delta y = 0 - \frac{1}{2}9.8(5)^2$$

$$\Delta y = -122.5 m$$

height of the building is 122.5 m

Example.2

An object, dropped from rest, falls down a distance of 45 m. What is the object's final velocity?

$$v^{2}_{f} = v^{2}i - 2g\Delta y$$
 $v^{2}_{f} = 0 - 2(9.8) (-45)$
 $v^{2}_{f} = 0 - 2(9.8) (-45)$
 $v_{f} = \pm 29.7 \text{ m/s}$
 $v_{f} = -29.7 \text{ m/s}$

1. An object is dropped from rest, and falls a distance of 20 m. What is the
object's final velocity?
2 An abject of mass 2 by is drawned (from most) from the slift of a building and
2. An object of mass 2 kg is dropped (from rest) from the cliff of a building and
it reached the ground after 7 seconds. What is the height of the building?
3. What is the velocity of a ball that has been dropped off a cliff after 5 seconds.
4. A ball is thrown vertically upward from the ground with a velocity of 20 m/s.
Calculate the maximum height reached.

5. A stone is dropped from rest from the top of a high building. After 2.00	s of
free-fall, a) what is the velocity of the stone? b) what is the displacement Δ	y of
the stone?	

Chapter 3 Newton's Laws

3.1 Newton's First Law

An external force causes a body to accelerate. If no external force(s) act on the body, the body will continue moving with a constant velocity along a straight line.

Force is vector quantity, with magnitude and direction. The force unit is Newton (N).

Mass is measured in kg (SI). Mass is a scalar quantity.

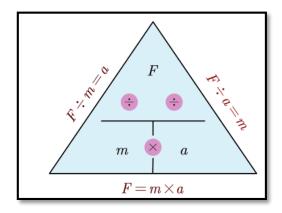
Formulas:

$$\sum \vec{F} = 0 \implies a = 0$$

3.2 Newton's Second Law

$$\sum \vec{F} = m \vec{a}$$

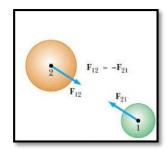
$$\Sigma F_x = ma_x \quad \Sigma F_y = ma_y \quad \Sigma F_z = ma_z$$



3.3 Newton's Third Law

The action and reaction forces act on different objects. The action force is equal in magnitude to the reaction force and is opposite in direction.

$$\vec{F}_{12} = -\vec{F}_{21}$$



Examples on Newton's Laws

- 1. The net force F acting on an object is equal to zero, if ...
- A) the object remains at rest
- B) the object is moving with constant acceleration
- C) the object moving in straight line with constant velocity
- D) a and c
- 2. When two bodies interact, they exert force on each other, and these forces are known as
- A) Ground-sky pair
- B) Air-water pair
- C) Action-reaction pair
- D) Friction-gravitation pair
- 3. A car, its mass is 1200 kg, moving with acceleration of magnitude 2 m/s². The net force on the car is.

Solution:

$$\Sigma F_x = ma_x$$

$$F_x = 1200 \times 2 = 2400 \text{ N}$$

4. A object has a mass of 900 kg and can accelerate from 0 to 27 m/s in 3 seconds. Calculate the net force needed to produce this acceleration.

Solution:

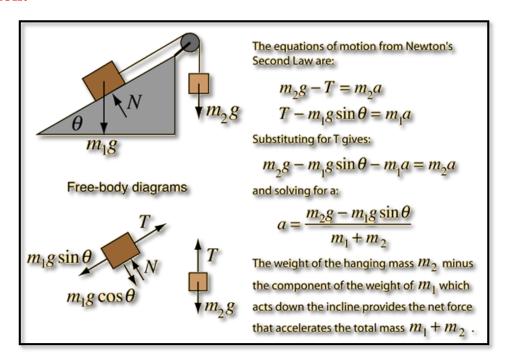
$$v - v_0 = at$$

$$a = \frac{v - v_0}{t}$$
 $a = \frac{27 - 0}{3} = 9 \frac{m}{s^2}$

$$\Sigma F_x = ma_x$$
$$F_x = 900 \times 9 = 8100 \, N$$

5. A body of mass m_1 rests on a smooth plane inclined to the horizontal. It is connected by a light inextensible string passing over a smooth pulley fixed at the top of the plane, to another body of mass m_2 hanging freely vertically below the pulley. Determine the acceleration of the system.

Solution:



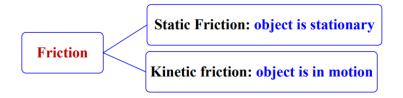
1. How can Newton's second law be represented as the second law between the second law betw	esented mathematically?

2. When two objects of unequal mass are hung vertically over frictionless pulley of negligible mass, the arrangement is called an Atwood machine as in the figure. Derive the formula for determining the magnitude of the acceleration of the two objects. 3. If the force acting on a body is 10 N, and the acceleration is 4 m/s², what is the mass of the body? 4. If the force acting on a body is 50 N, and the mass is 5 kg, what will be the acceleration of the body?

5. A constant force acting on a body of mass 3 kg changes its speed from 2 m/s to 3.5 m/s in 10 second. If the direction of motion of the body remains unchanged, what is the magnitude and direction of the force?		
6. Two forces F ₁ and F ₂ act on a 5.00		
kg object. Taking F_1 =20.0N and F_2 =15.0N, find the accelerations of the object for the configurations of forces shown in parts (a)		
and (b) of the figure.		
7. Two blocks of mass 4.0 kg and 6.0 kg are joined by a string and rest on a		
frictionless horizontal table (as in		
Figure). A force of 100 N is applied		

horizontally on one of the blocks. Find the acceleration of each block?			

3.4 Forces of Friction



Formulas.

Static friction	Kinetic friction	
$f_{s,\max} = \mu_s n$	$f_k = \mu_k n$	
$\mu_{\rm s}$ is the coefficient of kinetic friction	μ_k is the coefficient of kinetic friction.	
and n is the normal force	$\sum F_x = F - f_k$	

Example.1

A large block of ice is being pulled across a frozen lake. The block of ice has a mass of 300 kg. The coefficient of friction between two ice surfaces is small: $\mu_k = 0.05$. What is the force of friction that is acting on the block of ice?

Solution:

$$F_k = \mu_k n$$

n (normal force) = mg $F_k = \mu mg$

Substituting the values in the above equation we get,

$$F_k = 0.05 \times 300 \times 9.8$$

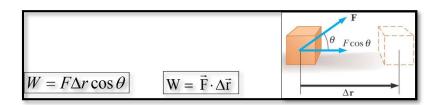
$$F_k = 147 \text{ kg.m/s}^2 \text{ or } 147 \text{ N}.$$

3. A force of 100 N is exerted on a 50 kg box and still at rest. If the coefficient of		
static friction is 0.3, calculate the static frictional force.		
4. A constant force of magnitude of 20 N is applied to move a body of mass 4 kg		
from rest on a rough horizontal surface with a friction force of 5 N. Calculate a)		
the coefficient of kinetic friction b) the acceleration of the body.		
5. A box slides down a 30° incline, with an acceleration of 1.2 m/s ² . Determine		
the coefficient of kinetic friction between the box and the incline.		

Chapter 4 Work and Energy

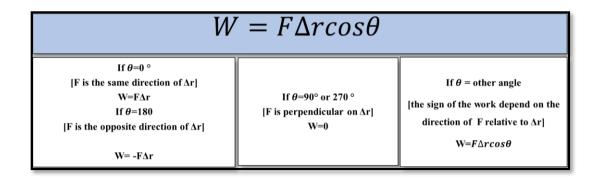
4.1 Work (W) Done by a Constant Force

Formulas:



Unit: N.m is called a joule (J)

F is the magnitude of the force, Δr is the magnitude of the object's displacement, and θ is the angle between F and Δr .



Example.1

Find the work done by a force of magnitude 50 N that pushed a particle 4 meter, knowing that the force is directed with 60° with particle's displacement.

$$W = F\Delta r \cos\theta$$

$$W = 50x4cos60 = 100J$$

Example.2

Calculate the work done when F = (5i+3j+2k) N and $\Delta r = (3i-j+2k)$ m acting in the same direction.

Solution:

$$W = \vec{F} \cdot \Delta \vec{r}$$

$$W = (5i+3j+2k). (3i-j+2k)$$

$$W = (5i+3j+2k). (3i-j+2k)$$

$$W = 15-3+4 = 16 J$$

1. Find the work done by a force of magnitude 40 N that caused a particle to move		
6 meters in the same direction of the force?		
2. A horizontal force F is applied to move a 5 kg carton across the floor. If the		
acceleration of the carton is measured to be 2 m/s ² . find work done by the force		
F to move the carton 7 m.		

3. A horizontal force F is applied to move a 6 kg carton across the floor. If the
carton starts from rest and its speed after 3 seconds is 6 m/s, how much work
does F do in moving the carton 4 m?

4.2 Kinetic Energy and the Work-Kinetic Energy Theorem

Kinetic Energy (KE): Energy associated with the motion of a body.

• Kinetic energy is a scalar quantity and it has the unit of (J).

$$KE = \frac{1}{2} mv^2$$

Example.1

A 7.00 kg object is moving with a velocity of 5.00 m/s. What is the kinetic energy of the object?

Solution:

K.E =
$$\frac{1}{2} \times m \times v^{2}$$

= $\frac{1}{2} \times 7 \times 5^{2}$
= 87.5 J

Example.2

What is the speed of an object of 1kg mass with a kinetic energy of 50J?

Solution:

$$KE = \frac{1}{2} mv^2$$

$$50 = \frac{1}{2} \times 1 \times v^2$$

$$v^2 = 100$$

$$v = 10 \text{ m/s}$$

Example. 3

The speed of an object of mass $10 \ kg$ increased from $5 \ m/s$ to $10 \ m/s$. Calculate the change in kinetic energy of the ball.

Solution:

Initial Kinetic Energy $KE_i = \frac{1}{2} \times m \times v_i^2$

$$KE_i = \frac{1}{2} \times 10 \times (5)^2 = 125 \text{ J}$$

 $KE_i = 125 \text{ J}$

Finial Kinetic Energy $KE_f = \frac{1}{2} \times m \times v_f^2$

$$KE_i = \frac{1}{2} \times 10 \times (10)^2 = 500 \text{ J}$$

Change in kinetic energy = 500-125 = 375 J

Practice Questions

1. Calculate the kinetic energy of an object of mass 2 kg, moving with a speed of 6 m/s.

2. An object with 2 kg is moving at 4 m/s. How much kinetic energy does it have?				
3. A cube of mass 10 kg slows down from a speed of 10 m/s to 5 m/s. Calculate the				
change in kinetic energy of the cube.				

4.3 Kinetic Energy and the Work–Kinetic Energy Theorem

Formulas:

$$\sum W = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2$$

$$\sum W = K_f - K_i = \Delta K E$$

Example.1

A boy pulls a 6.0 kg object initially at rest on a smooth floor with a constant horizontal force of 12N. Compute the change in kinetic energy of the box after the boy has pulled it a distance of 3.0 m.

$$W = KE_f - KE_I$$

$$= \frac{1}{2}mv_f^2 - 0$$

$$v_f = \sqrt{\frac{2W}{m}}$$

$$= \sqrt{\frac{2F\Delta x}{m}} = 3.5 \text{ m/s}$$

Example.2

An 8 kg block initially at rest is pulled to the right for 3m with a force of 12N over a frictionless surface. Determine its final speed.

Solution:

$$W = KE_f - KE_I$$

$$= \frac{1}{2} m v_f^2 - 0$$

$$v_f = \sqrt{\frac{2W}{m}}$$

$$= \sqrt{\frac{2F\Delta x}{m}} = 9 \text{ m/s}$$

. An 10 kg block initially at rest is pulled to the right for 5m with a force of
20N on a frictionless horizontal surface. Determine the final speed of the block.
2. A 2.0 kg block is initially moving at a speed of 5.0 m/s on a rough horizontal
surface, with $\mu_K = 0.5$. Calculate the speed of the block after moving 2.0 m.

3. A block of mass 5kg is initially at rest on a rough horizontal surface. A force
of 45N acts on it in a horizontal direction and pushes the block over a distance
of 2m. The force of friction acting on the block is 25N. find the final kinetic
energy of the block.

4.4 Potential Energy of a System

The gravitational potential energy U_g

 U_g depends only on the <u>vertical height</u> of the object above the surface of the Earth. It has the unit <u>joule (J)</u>, and it is a <u>scalar</u> quantity.

Formulas:

$$U_g = mgy$$

$$\Delta PE = \Delta U_g = mgy_f - mgy_i$$

Work due to gravity is given by: $W = -\Delta PE$.

Therefore, for a free fall motion along the y-direction, and ignoring air resistance:

$$\Delta KE = mgy_i - mgy_f$$

Example.1

A mass of 2 kg is thrown upward from the ground to the height of 10 m. Find the potential energy of the object.

Solution:

$$\Delta PE = mg\Delta y$$
$$= 2 \times 9.8 \times 10 = 196 J$$

Example.2

A box has a mass of 8 kg. The box is lifted from the garage floor and placed on a shelf. If the box gains 200 J of Potential Energy, how high is the shelf?

$$\Delta PE = mg\Delta y$$

$$\Delta y = \frac{\Delta PE}{mg} = \frac{200}{8 \times 9.8} = 2.55 \, m$$

. A mass of 5 kg is thrown upward from the ground to a height of 100 m. Find
he potential energy of the object.
2. A man climbs onto a wall that is 3.6m high and gains 2268 J of potential energy.
What is the mass of the man?

3. A box has a mass of 3 kg. The box is lifted from the gai	rage floor and placed
on a shelf. If the box gains 150 J of Potential Energy, how h	nigh is the shelf.

4.5 Conservation of mechanical energy

The sum of kinetic and potential energies is $\underline{\mathbf{mechanical\ energy}}$ (E_{mec})

Formulas:

$$E_{mec} = KE + U_g$$

$$E_f = E_i$$

$$K_f + U_f = K_i + U_i$$

$$\frac{1}{2}mv_f^2 + mgy_f = \frac{1}{2}mv_i^2 + mgy_i$$

Example.1

A diver of mass m drops from a board 10.0 m above the water's surface. Neglecting air resistance, (a) Find his speed 5.0 m above the water surface (b) Find his speed as he hits the water.

$$y_i = 10 \text{ m}$$

(a) his speed 5.0 m above the water surface

$$\frac{1}{2}mv_i^2 + mgy_i = \frac{1}{2}mv_f^2 + mgy_f$$

$$0 + gy_i = \frac{1}{2}v_f^2 + gy_f$$

$$v_f = \sqrt{2g(y_i - y_f)}$$

$$= \sqrt{2(9.8m/s^2)(10m - 5m)} = 9.9m/s$$

(b) his speed as he hits the water

$$0 + mgy_i = \frac{1}{2}mv_f^2 + 0 \qquad v_f = \sqrt{2gy_i} = 14m/s$$

Example.2

A ball is initially at rest and is dropped from a height of y_i (m). Derive the formula of the velocity of the ball when it reaches the ground.

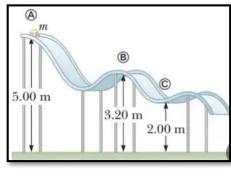
$$\frac{1}{2}mv_{i}^{2} + mgy_{i} = \frac{1}{2}mv_{f}^{2} + mgy_{f}$$

$$0 + gy_{i} = \frac{1}{2}v_{f}^{2} + 0$$

$$v_{f} = \sqrt{2g(y_{i})}$$

Practice Questions.

1. A block of mass m= 10 kg is released from rest at point A and slides on the frictionless track shown in Figure. Determine the block's speed at points B and C?



2. A swimmer with 72.0 kg jumps into a swimming pool from 3.25 m above the
water. Use energy conservation to find his speed when he just hits the water.

3. The box in the figure was pushed to the right with an initial speed of $8\ m/s$.				
Find the speed of the box at point B.	$v_i = 8 m/s$ 2.2 m B			
4. A skier of mass 60kg at rest slides dow	on a slope. The length along the slope is			
100 m and the skier loses a vertical height of 50 m. If friction is negligible, what				
is the final velocity of the skier at the lower end of the slope?				

Chapter 5 Electric field

5.1 Electric Charges.

- There are two kinds of charges: positive and negative.
- Charges of the same signs repel one another and Charges of opposite signs attract one another. The unit of charge is coulomb (C).
- Electric charge is quantized $q=\pm Ne \mbox{ where } N=1,2,.... \mbox{ and } e=e=1.6\times 10^{-19}\mbox{ C (the charge of one}$

Example.1

electron)

- 1. Charges of the same sign ______one another and charges with opposite signs _____ one another.
- A) Repel, repel B) **Repel, attract** C) Attract, repel D) Attract, attract

Example.2

An object has an excess charge of -3.84×10^{-17} C. How many excess electrons does it have? The electron charge is -1.6×10^{-19} C.

$$q = \pm Ne$$

$$N = \frac{q}{e} = \frac{-3.84 \times 10^{-17}}{-1.6 \times x10^{-19}} = 120$$

Prac	tice	Oı	iesti	ions
		ч,		~

1. What is the total charge of 8.23×10^{31} of electrons?
2. An object has an excess charge of -7.68×10^{-17} C. How many excess electrons does it have? The electron charge is -1.6×10^{-19} C.
3. How many electrons need to be removed from a penny to leave it with a charge of $+1.0\times10^{-7}$ C?

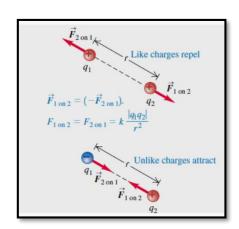
5.2 Coulomb's Law

The magnitude of the electric force (F_e)

Formulas:

$$F_e = k_e \, rac{|\,q_1|\,|\,q_2|}{r^2}$$

r is a separation between the particles and q_1 and q_2 is the charges of the particles. $k_e = 8.987 \ 5 \ x \ 10^9 \ N.m^2/C^2$ (Coulomb constant)



Example.1

A point charge of $+3.00 \times 10^{-6}$ C is 12.0 cm distant from a second point charge of -1.50×10^{-6} C. Calculate the magnitude of the force on each charge.

Solution:

$$F = k \frac{|q_1 q_2|}{r^2}$$

$$= (8.99 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}) \frac{(3.00 \times 10^{-6} \text{ C})(1.50 \times 10^{-6} \text{ C})}{(12.0 \times 10^{-2} \text{ m})^2} = 2.81 \text{ N}$$

Example.2

What must be the distance between point charge $q_1 = 26.0 \mu C$ and point charge $q_2 = -47.0 \mu C$ for the electrostatic force between them to have a magnitude of 5.70 N?

Solution:

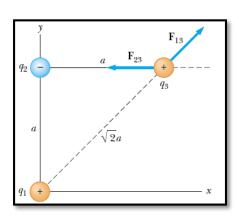
$$F = k \frac{|q_1 q_2|}{r^2} \qquad \Longrightarrow \qquad r^2 = k \frac{|q_1 q_2|}{F}$$

$$r^2 = (8.99 \times 10^9 \, \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}) \frac{(26.0 \times 10^{-6} \, \text{C})(47.0 \times 10^{-6} \, \text{C})}{(5.70 \, \text{N})} = 1.93 \, \text{m}^2$$

$$r = \sqrt{1.93 \,\mathrm{m}^2} = 1.39 \,\mathrm{m}$$

Example.3

Consider three point charges located at the corners of a right triangle as shown in Figure 23.7, where $q_1 = q_3 = 5.0 \,\mu\text{C}$ and $a = 0.10 \,\text{m}$. Find the resultant force exerted on q_3 .



find the magnitude of
$$\vec{\mathbf{F}}_{23}$$
:
$$F_{23} = k_{\epsilon} \frac{|q_2| |q_3|}{a^2}$$

=
$$(8.988 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \frac{(2.00 \times 10^{-6} \text{ C})(5.00 \times 10^{-6} \text{ C})}{(0.100 \text{ m})^2} = 8.99 \text{ N}$$

Find the magnitude of the force
$$\vec{\mathbf{F}}_{13}$$
: $F_{13} = k_{\epsilon} \frac{|q_1| |q_3|}{(\sqrt{2} a)^2}$

=
$$(8.988 \times 10^9 \,\mathrm{N \cdot m^2/C^2}) \frac{(5.00 \times 10^{-6} \,\mathrm{C})(5.00 \times 10^{-6} \,\mathrm{C})}{2(0.100 \,\mathrm{m})^2} = 11.2 \,\mathrm{N}$$

Find the x and y components of the force
$$\vec{\mathbf{F}}_{13}$$
: $F_{13x} = (11.2 \text{ N}) \cos 45.0^{\circ} = 7.94 \text{ N}$

$$F_{13y} = (11.2 \text{ N}) \sin 45.0^\circ = 7.94 \text{ N}$$

Find the components of the resultant force acting on
$$q_3$$
: $F_{3x} = F_{13x} + F_{23x} = 7.94 \text{ N} + (-8.99 \text{ N}) = -1.04 \text{ N}$

$$F_{3y} = F_{13y} + F_{23y} = 7.94 \text{ N} + 0 = 7.94 \text{ N}$$

Express the resultant force acting on
$$q_3$$
 in unit-vector form:

$$\vec{\mathbf{F}}_3 = (-1.04\hat{\mathbf{i}} + 7.94\hat{\mathbf{j}}) \,\mathrm{N}$$

1. The magnitude of electron charge is 1.6×10^{-9} C and of proton charge is also 1.6×10^{-19} C. In a hydrogen atom consisting of one electron and one proton separated by a distance of approximately 5.3×10^{-11} m, calculate the magnitude of the electric force.
2. Two point charges of 2 nC, -1 nC are separated by a distance of 1m. Find the the magnitude of electric force between the two charges.
3. Three point charges lie along a straight line as shown in the figure below, where q_1 = 6.36 μ C, q_2 =1.63 μ C, and q_3 = -1.98 μ C. The separation distances are d_1 =0.03m and d_2 =0.02m. Calculate the magnitude and direction of the net
electric force on each of the charges.

5.3 Electric field (E)

Vector quantity and its unit N/C

Formulas:

Electric field (E)	N/C
$\mathbf{E} \equiv \frac{\mathbf{F}_e}{q_0}$ $\mathbf{F}_e = q\mathbf{E}$	Electric field is the electric force F_e acting on a positive test charge q_0 placed at that point divided by the test charge:
$\mathbf{E} = k_e \frac{q}{r^2} \hat{\mathbf{r}}$	Electric field due to a point charge
$\mathbf{E} = k_e \sum_{i} \frac{q_i}{r_i^2} \hat{\mathbf{r}}_i$	Electric field due to a group of point charges

Example.1

Calculate the electric field strength acting on a 4 μ C charge if the electric force on this charge was 20 $\times 10^3$ N.

Solution:

$$E = F_e/q$$

$$= \frac{20 \times 10^3}{4 \times 10^{-9}} = 5 \times 10^9 \, N/C$$

Example.2

Find the magnitude of electric field strength due to a charge of 5 µC at 80 cm.

Solution:

$$\mathbf{E} = k_e \frac{q}{r^2} \hat{\mathbf{r}}$$

$$E = 9 \times 10^9 \times \frac{5 \times 10^{-6}}{(80 \times 10^{-2})^2} = 8 \times 10^{-4} \text{ N/C}$$

Example.3

The electric field midway between two charges of +3q and -2q is 98.0 N/C and the distance between the charges is 0.2 m. What is the value of the q?

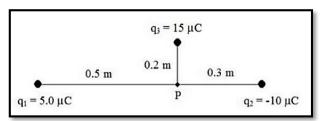
$$E = E_1 + E_2$$

$$98 = \frac{K(3q)}{(0.1)^2} + \frac{K(2q)}{(0.1)^2}$$

$$98 = \frac{K.5q}{0.01} \Rightarrow q = \frac{98 \times 0.01}{5 \times 9 \times 10^9} = 2.177 \times 10^{-11} \text{C}$$

1. Calculate the strength and direction of the electric field due to a point charge of 2.00
nC (nano-Coulombs) at a distance of 5.00 mm from the charge.
2. Two positive charges of $1\mu C$ and $2\mu C$ are placed 1 m apart. Calculate the value of
the electric field in N/C in the middle point of the line joining the axis.
3. Calculate the electric field strength acting on a 10 μ C charge if the electric
force on this charge was 100 x10 ³ N.

4. Three-point charges are fixed in place as shown in the figure below. Determine the magnitude of the electric field at point P.



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••••••	•••••	 ••••••	•••••
••••••	•••••	 •••••	•••••

5.4 Electric potential (V)

Electric potential (V) is the amount of work needed to move a unit charge from a reference point to a specific point against an electric field. V is a scalar quantity. SI unit of both V, and potential difference (ΔV) is (V) J/C

5.4.1 Potential Differences in a Uniform Electric Field

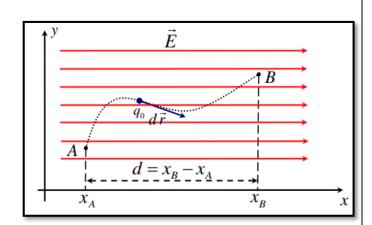
Formulas:

$$\Delta V = V_B - V_A = -Ed$$

The change in potential energy of the charge-field system is

$$\Delta U = q_0 \Delta V = -q_0 E d$$





A proton is released from rest in a uniform electric field that has a magnitude of $8.0 \, 10^4 \, \text{V/m}$ (Fig. 25.6). The proton undergoes a displacement of 0.50 m in the direction of E.

- (A) Find the change in electric potential between points A and B.
- (b) Find the change in potential energy of the proton-field system for this displacement

Solution:

(A)
$$\Delta V = -Ed = -(8.0 \times 10^4 \,\text{V/m}) \,(0.50 \,\text{m}) = -4.0 \times 10^4 \,\text{V}$$

(B)
$$\Delta U = q_0 \ \Delta V = e \ \Delta V$$
$$= (1.6 \times 10^{-19} \ \text{C}) (-4.0 \times 10^4 \ \text{V})$$
$$= -6.4 \times 10^{-15} \ \text{J}$$

- 1. Two large parallel metal plates are $5.0 \ cm$ apart. The magnitude of the electric field between them is $800 \ N/C$.
- (a) What is the potential difference between the plates?

(b) What work is done when one electron is moved from the positive to the negative
plate?

2. A voltmeter measures the potential difference between two large parallel plate
to be 50.0 volts. The plates are 3.0 cm apart. What is the magnitude of the electri
field strength between the plates?

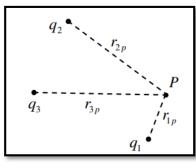
3. The electric potential difference between the ground and a cloud in a particular thunderstorm is 1.2×10^9 V. What is the magnitude of the change in energy of an electron that moves between the ground and the cloud?

5.4.2 Electric potential created by a point charge q at any distance r from the charge.

Formulas:

$$V = k_e \frac{q}{r}$$

$$V_p = k_e \frac{q_1}{r_{1p}} + k_e \frac{q_2}{r_{2p}} + k_e \frac{q_3}{r_{3p}}$$



The potential difference between two arbitrary points A and B

$$V_B - V_A = k_e q_1 \left[\frac{1}{r_B} - \frac{1}{r_A} \right]$$

Example.1

(a) Find the potential at a distance of 1.00 cm from a proton. (b) What is the potential difference between two points that are 1.00 cm and 2.00 cm from a proton?

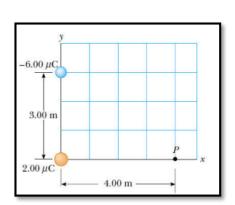
Solution:

- (a) The potential at 1.00 cm is $V_1 = k_e \frac{q}{r} = \frac{\left(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2\right) \left(1.60 \times 10^{-19} \text{ C}\right)}{1.00 \times 10^{-2} \text{ m}} = \boxed{1.44 \times 10^{-7} \text{ V}}$
- (b) The potential at 2.00 cm is $V_2 = k_e \frac{q}{r} = \frac{\left(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2\right) \left(1.60 \times 10^{-19} \text{ C}\right)}{2.00 \times 10^{-2} \text{ m}} = 0.719 \times 10^{-7} \text{ V}$. Thus, the difference in potential between the two points is $\Delta V = V_2 V_1 = \boxed{-7.19 \times 10^{-8} \text{ V}}$.

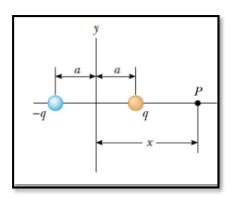
Practice Questions.

1. Calculate the electric potential of a $2.0 \ nC$ point charge at a distance of $5.0 \ cm$ from the charge.

2. A charge $q_1 = 2 \mu C$ is located at the origin, and a charge $q_2 = -6 \mu C$ is located at (0, 3) m, as shown in Figure. Find the total electric potential due to these charges at point P, whose coordinates are (4,0) m



3. An electric dipole consists of two charges of equal magnitude and opposite sign separated by a distance 2a, as shown in the Figure. The dipole is along the x axis and is centred at the origin. Calculate the electric potential at point P.



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5.5 Current and Resistance.

Formulas:

average current (I_{av})	instantaneous current (I)	
Unit is ampere (A): 1 C/s	Unit is ampere (A): 1 C/s	
Q is the amount of charge		
$I_{av} = \frac{\Delta Q}{\Delta t}$	$I \equiv \frac{dQ}{dt}$	

Example.1

If the current in an electric circuit is 1mA, how much charge flows through the circuit in 10 second?

1		4 .			
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$$I_{av} = \frac{\Delta Q}{\Delta t}$$
$$1 \times 10^{-3} = Q/10$$
$$Q = 10^{-2} C$$

1. A 2 min long cross-section of whe is isolated and 20 °C of charge passed
through the wire in 40 s. Calculate the average current.
2. Calculate the current through a lamp when a charge of 4 C passes through it in
500 s.

Formulas

V=IR	V=voltage (V) I = current (A) R =resistance (Ω)
Ohm law	
$R = \rho \frac{\ell}{A}$	ρ is resistivity in $(\Omega \cdot m)$, l is the length of the wire and A is the cross-section
Resistance	area of the wire (m ²)
$ \rho = \frac{1}{\sigma} $	σ is Conductivity in Siemens per meter (S/m)
$P = I\Delta V, \qquad P = I^2 R, \qquad P = \frac{\Delta V^2}{R}$	Watt (W)
P is the Electrical Power	

Example.1

The plate of a certain steam iron states that the iron carries a current of 7.40 A when connected to a 120 V source. What is the resistance of the steam iron?

Solution:

$$R = \frac{\Delta V}{I} = \frac{120}{7.4} = 16.2 \,\Omega$$

Example.2

If 750 μA is flowing through 11 k Ω of resistance, what is the voltage drop across the resistor?

$$\Delta V = I R = 750 \times 10^{-6} \times 11 \times 10^{3}$$

= 8.25 V

Example.3

Calculate the resistance of an aluminium cylinder that is 10 cm long and has a cross sectional area of $2x10^{-4}$ m² having a resistivity of $2.82x10^{-8}\Omega$.m. Repeat the calculation for a cylinder of the same dimensions and made of glass having a resistivity of $3x10^{10}\Omega$.m.

Solution:

$$R=\rho\frac{\ell}{A}=\left(2.82\times10^{-8}\Omega\cdot\mathrm{m}\right)\left(\tfrac{0.100\,\mathrm{m}}{2.00\times10^{-4}\,\mathrm{m}^2}\right)=1.41\times10^{-5}\Omega$$
 Similarly, for glass we find that

$$R =
ho rac{\ell}{A} = \left(3.0 imes 10^{10} \Omega \cdot ext{m}
ight) \left(rac{0.100 ext{ m}}{2.00 imes 10^{-4} ext{ m}^2}
ight) = 1.5 imes 10^{13} \Omega$$

Example.4

A light bulb operating at a voltage of 100 V has a resistance of 200 Ω . How much power is dissipated in this bulb?

Solution:

$$P = \frac{\Delta V^2}{R} = \frac{100^2}{200} = 50 W$$

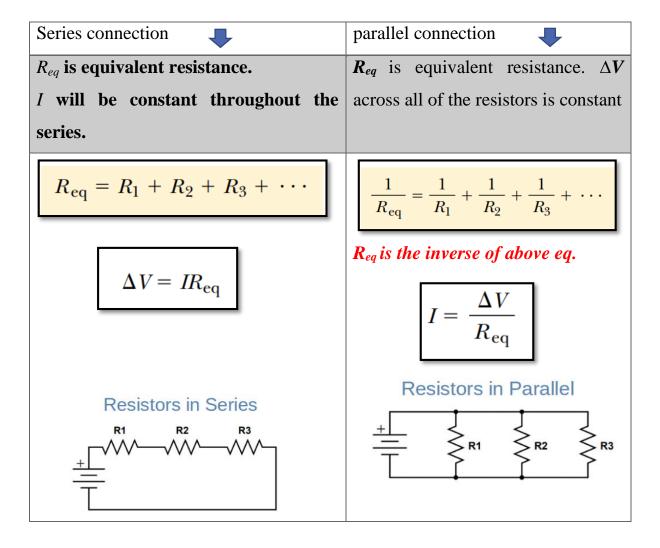
1. What is the resistance and current through a 100-W light if connected to a 120
V source?

2. A cylindrical aluminium wire is 200×10^{-3} m long and has a cross-sectional area
of $100 \times 10^{-4} \text{m}^2$ and a resistivity (ρ) of $2.8 \times 10^{-8} \ \Omega$.m. If a potential difference of
100 V is applied across the wire, find the following:
a) The resistance of the wire (R) .
b) The current in the wire (I)
c) The conductivity of aluminium (σ) .
3. A 200 W light bulb is connected across a 100 V dc power supply. What current
will flow through this bulb?

4. Using Ohm's law, find out the current through 25.0 $\mu\Omega$ resistor when it is
connected across a 100 V dc power supply.
5. A resistor is constructed of a carbon rod that has a uniform cross-sectional area
A=5 mm ² . When a potential difference $V=15$ V is applied across the ends of the
rod, there is a current $I = 4 \times 10^{-3} \text{A}$ passing through the rod. The resistivity of
carbon $\rho=3.5 \times 10^{-5} \Omega$.m. Find:
a) the resistance of the rod
a) the resistance of the rod
b) the rod's length.
6. A copper wire has a length of 160 m and a diameter of 1.00 mm. If the wire is
connected to a 1.5-volt battery, how much current flows through the wire?
Knowing that ρ of copper = 1.72 ×10 ⁻⁸ Ω .m.

5.5.1 Resistor Connection

Formulas:



Example.1

Three resistors are connected in parallel. A potential difference of 18.0 V is maintained between points a and b. Calculate the equivalent resistance of the three resistors, and then calculate the electric current passing through each resistance.

Solution:

$$\frac{1}{R_{\rm eq}} = \frac{1}{3.00 \,\Omega} + \frac{1}{6.00 \,\Omega} + \frac{1}{9.00 \,\Omega}$$

$$R_{\rm eq} = \frac{18.0 \,\Omega}{11.0} = 1.64 \,\Omega$$

$$I_1 = \frac{\Delta V}{R_1} = \frac{18.0 \text{ V}}{3.00 \Omega} = 6.00 \text{ A}$$

$$I_2 = \frac{\Delta V}{R_2} = \frac{18.0 \text{ V}}{6.00 \Omega} = 3.00 \text{ A}$$

$$I_3 = \frac{\Delta V}{R_3} = \frac{18.0 \text{ V}}{9.00 \Omega} = 2.00 \text{ A}$$

1. Two resistors of 2Ω and 3Ω are connected in series, with a battery of 6.0 V.
calculate the equivalent resistance and the current through the circuit.
2. Two resistors 3 Ω and 2 Ω are connected in parallel and a potential difference
of 12 V is connected across them. Find:
(A) The equivalent resistance of the parallel combination of the resistors.

(B) The total current through the circuit.	
(C) The current through each branch.	
3. a) Find the equivalent resistance between point potential difference of 34.0 V is applied between	
current in each resistor.	$ \begin{array}{c} 7.00 \Omega \\ 4.00 \Omega \\ \hline 10.0 \Omega \end{array} $

5.6 Capacitors

A capacitor consists of two conductors separated by an insulator and carrying charges of equal magnitude but opposite sign $(+Q \ and \ -Q)$.



Capacitance (C) is the capability of a material object or device to store electric charge. C is always positive. The SI unit of capacitance is **Farad** (F or C/V).

Fomulas:

$$C = \frac{Q}{V}$$

C for parallel plate capacitors

$$C = \frac{\varepsilon A}{d}$$

Where.

C = Capacitance in Farads

 ε = Permittivity of dielectric (absolute, not relative)

A = Area of plate overlap in square meters

d = Distance between plates in meters

Example.1

If the maximum amount of charge held by a capacitor at a voltage of 12V is 36-C, what is the capacitance of this capacitor?

$$C = \frac{Q}{V}$$

$$C = \frac{36C}{12V} = 3F$$

Example.2

How much charge is stored on each plate of a 4.00 μ F capacitor when it is connected to a 12.0 V battery?

Solution:

$$Q = CV$$

$$= 12x4 \times 10^{-6}$$

$$Q = 48\mu F$$

Example.3

A parallel-plate capacitor with air between the plates has an area $A = 2.00 \times 10^{-4}$ m² and a plate separation d = 1.00 mm. Find its capacitance

Solution:

$$C = \frac{\epsilon_0 A}{d} = \frac{(8.85 \times 10^{-12} \,\mathrm{C}^2/\mathrm{N \cdot m}^2) (2.00 \times 10^{-4} \,\mathrm{m}^2)}{1.00 \times 10^{-3} \,\mathrm{m}}$$
$$= 1.77 \times 10^{-12} \,\mathrm{F} = \boxed{1.77 \,\mathrm{pF}}$$

1. If the maximum amount of charge held by a capacitor at a voltage of 10 V is 10
μC , what is the capacitance of this capacitor?

ıC	a 12.0 V battery?
•••	
3.	. Two conducting parallel plates, having net charges of +10 μ C and -10 μ C, ha
a	potential difference of 10 V between them.
a)	Determine the capacitance of the system.
,	
•••	
•••	
ړ ل	What is the notantial difference between the two conductors if the abordes
	What is the potential difference between the two conductors if the charges
ea	ach side of the capacitor are increased to +100 μC and -100 μC?
•••	
4.	. A capacitor consists of two parallel plates, each with an area of 7.60 cm
	eparated by a distance of 1.80×10 ⁻³ m. A 20.0 V potential difference is appli

a) The capacitance of the capacitor,
b) The charge on each plate.
5. If the maximum amount of charge held by a capacitor at a voltage of 6 V is 0.6
C, what is the capacitance of this capacitor?

5.6.1 Capacitors Connection

Formulas:

Series connection	parallel connection
C_{eq} is the equivalent capacitance	C_{eq} is the equivalent capacitance
Q (the charge) is the same on all of the	ΔV (the potential difference) is the same
capacitors	across all of capacitors

$$\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots$$

$$C_{\text{eq}} = C_1 + C_2 + C_3 + \cdots$$

C_{eq} is the inverse of above eq.

$$\Delta V = \Delta V_1 + \Delta V_2$$

$$\Delta V = Q \frac{1}{C_{eq}}$$

$$Q_1 = Q_2 = Q$$

$$C_1$$

$$\Delta V_1$$

$$\Delta V_2$$

$$\Delta V_2$$

$$Q = Q_1 + Q_2$$

$$Q = C_{eq} \Delta V$$

$$Q_1$$

$$Q_2$$

$$Q_2$$

Example.1

Find the total capacitance for two capacitors connected in series, given that their individual capacitances are 1.0 and $5.0 \,\mu F$.

Solution:

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$
$$\frac{1}{C_{eq}} = \frac{1}{1.0} + \frac{1}{5.0} + \frac{1}{8.0} = \frac{1.20}{\mu F}$$

Inverting gives

$$C_{eq} = 0.83 \, \mu F$$

Example.2

Three capacitors each of capacitance $1\mu F$ are connected in parallel. To this combination, a fourth capacitor of capacitance $6\mu F$ is connected in series. The resultant capacity of the system is:

Solution:

For parallel

$$C_{eq-p} = C_1 + C_2 + C_3 = 3 \ \mu F$$

For series

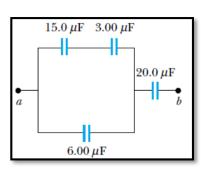
$$\frac{1}{C_{eq}} = \frac{1}{C_{eq-p}} + \frac{1}{6} = \frac{1}{3} + \frac{1}{6}$$

Inverting gives

$$C_{eq} = 2 \mu F$$

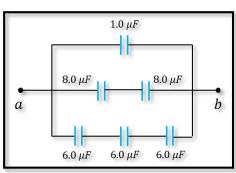
1. You have 3 capacitors in series. Their capacitances are 4 μF , 3 μF , and 2 μF .
What is the total capacitance of the system?
2. You have 4 capacitors in parallel. Their capacitances are 5 μF , 3 μF , 2 μF ,
and 1 μF . What is the total capacitance of the system?

3. Four capacitors are connected as shown in figure below. Find the equivalent capacitance between points a and b.



4. You have 3 capacitors in series. Their capacitances are 4 μF , 3 μF , and 2 μF . What is the total capacitance of the system?

5. Six capacitors are connected as shown in the following figure. (a) Find the equivalent capacitance between points a and b and (b) Find the total charge of the system if ΔV_{ab} =



20 V.

5.6.2 Energy Stored in a charged Capacitor.

Since capacitors store electric charge, they store electric potential energy U.

Unit of U is Joule (J)

Formulas

$$U = \frac{1}{2}Q \Delta V \qquad \qquad U = \frac{Q^2}{2C} \qquad \qquad U = \frac{1}{2}C(\Delta V)^2$$

Example.1

(a) A $3.00-\mu F$ capacitor is connected to a 12.0-V battery. How much energy is stored in the capacitor? (b) If the capacitor had been connected to a 6.00-V battery, how much energy would have been stored?

Solution:

(a)
$$U = \frac{1}{2}C(\Delta V)^2 = \frac{1}{2}(3.00 \ \mu\text{F})(12.0 \ \text{V})^2 = \boxed{216 \ \mu\text{J}}$$

(b)
$$U = \frac{1}{2}C(\Delta V)^2 = \frac{1}{2}(3.00 \ \mu\text{F})(6.00 \ \text{V})^2 = \boxed{54.0 \ \mu\text{J}}$$

Example.2

Assume that an energy of 300 J is to be delivered from a 30.0- μ F capacitor. To what potential difference must it be charged?

$$U = \frac{1}{2}C\Delta V^{2}$$

$$\Delta V = \sqrt{\frac{2U}{C}} = \sqrt{\frac{2(300 \text{ J})}{30 \times 10^{-6} \text{ C/V}}} = \boxed{4.47 \times 10^{3} \text{ V}}$$

1. Calculate the energy stored in 2 mF capacitor charged to a potential difference
of 2 V?
2. If the charge stored in a capacitor is $4 nC$ and the value of capacitance is $2 nF$,
calculate the energy stored in the capacitor.
3. If the charge in a capacitor is 4C and the energy stored in it is 4J, calculate the
voltage across its plates.
vertuge meress are printes.
4. Calculate the charge and energy stored in a capacitor of capacitance 32 μF ,
when it is charged to a potential difference of 0.6 kV.
when it is charged to a potential difference of 0.0 kV.