Program Guide

Master of Science Program in Mathematics

The Department of Mathematics at Al-Imam Mohammad Ibn Saud Islamic University, Riyadh was established with the objective that its undergraduate students upon graduation accommodate to the consistently growing demand of the job market in the Kingdom of Saudi Arabia and to effectively contribute to higher education system of the country. Since then the department has been offering B.Sc. degree, a large number of students have completed their B.Sc in mathematical Sciences from the department. Now it is the suitable time for the department to enhance its fulfillments of the set objectives by offering a Master's degree Program to start hopefully in September, 2014.

1. Program's Mission Statement:

To prepare well qualified staff who will contribute effectively in economic and social developments of Saudi Arabia and who will work innovatively on enhancing the higher education system of the country in the field of mathematics and its applications to other disciplines.

2. Program's Objectives:

The M.Sc. Program in Mathematics targets at the following objectives:

a) Enabling students of the program to exploit their abilities and potential to effectively enhance their mathematical skills.

b) Providing the program's students with appropriate skills to become independent learners and be experienced in doing scientific researches.

c) Providing students with the necessary knowledge and preparation needed to continue their higher studies leading to PhD degree in mathematics or related fields at local and international universities.

3. Admission Requirements:

In addition to conditions mentioned in article (13) of the **(UGSP), a**pplicants to the Master Program should fulfill the followings:

a) The applicant should have a B.Sc. degree in mathematics from an official local university or a recognized international university with a GPA equals or equivalent to 3.75 out of 5.

b) The applicant should pass the entry exam set by the Mathematics Graduate Committee **(MGC)**.

c) The applicant should get a TOEFL score at least 400 or equivalent scores in other recognized international English tests.

d) An applicant who was admitted for this program with his(her) B.Sc in mathematics from a college other than the College of Science may be required to finish successfully some complementary undergraduate courses before registering any course of the master program.

e) The complementary undergraduate courses mentioned in (d) are determined for each student by the Mathematics Graduate Committee and should be taken within three academic semesters from his(her) enrollment in the program and being dealt on that according to article (18) of **(UGSP).**

f) Students who are enrolled in another recognized mathematics master program having at least a GPA equal or equivalent to 3.75 out of 5 may be transferred to the program upon establishing all related conditions mentioned above and upon fulfilling the requirements mentioned in article (30) of **(UGSP).**

g) Applicants who got a B.Sc in a scientific major other than mathematics will be dealt case by case and an appropriate decision for them will be made.

h) Students who are enrolled in another graduate program in Al-Imam University or in another recognized master program other than mathematics will be dealt case by case and an appropriate decision for them will be made according to article (31) of **(UGSP).**

4. Program Regulations:

In addition to the (UGSP) regulations mentioned in articles (20)-(23) and articles (32)-(40), the following regulations apply:

a) A student admitted to the master program should be assigned an academic supervisor by the **(MGC)** upon starting the program to guide and help him(her) throughout his(her) academic program.

b) Number of credit hours for the master program is forty four credit hours to be worked out normally over four semesters in two academic years.

c) The program consists of ten courses - four credit hours for each one - and a research project course of four credit hours.

d) The program's compulsory core courses are six courses mentioned in item **(6-I)** and the remaining four courses are electives courses for which the student chooses either from lists A or from list B mentioned in item **(6-II)** upon consent of his(her) academic supervisor.

e) Throughout the program, no more than three courses can be registered by a student in each semester. Exemption of that condition may be made upon the recommendation of student's advisor and the approval of (MGC).

f) No elective course of the program can be taken before a compulsory core course.

g) Student with special needs or conditions may be allowed to take only two courses in a semester upon the consent of supervisor and **(MGC)**.

h) The passing score for any course in the master program – except the research project course – is 70 of 100 scores or higher.

i) The research project course passing score is 80 of 100 scores or higher, otherwise it will be classified as incomplete.

j) The research project course must be completed within three semesters of its registration.

k) If a student failed a course with score of 50 scores or higher, he(she) is allowed to take a makeup exam of 100 points during the next semester. No more than one course in a semester is allowed for makeup exam.

l) The passed makeup exams are graded by C grade regardless of the actual obtained scores.

m) Transferring students may be exempted of at most three courses of the master program's courses not including the research project course. Any exempted course should be passed with a grade of at least 4 out of 5 or equivalent.

n) A student registering three courses or more in a semester may be allowed to drop at most one course upon the consent of his(her) supervisor and the approval of **(MGC)**.

o) The Program's student may not drop more than three courses throughout his(her) entire program.

p) Every student of the master program who finished the core courses should determine the topic of his(her) research project upon the beginning of next semester; after then a project advisor will be assigned for him(her) by **(MGC)**.

q) The teaching and research language is English.

5. Termination from the Master Program:

In addition to termination cases mentioned in articles (25), (26) of **(UGSP)**, a program's student will be terminated upon satisfying one of the followings:

a) Failing to pass three courses throughout the study program.

b) Failing to pass two courses in one semester.

c) Failing to pass same course twice.

d) Failing to complete the research project course within three semesters from its registration.

e) Obtaining GPA less than 3.75 in two consecutive semesters or three nonconsecutive semesters throughout the program.

f) Exemption of termination and extension of the allowed time period of the master program are dealt according to regulations mentioned in articles (27) - (29) of (**UGSP).**

6. List of Courses:

I. Core Courses:

1. MAT 511: Measure & Integration
2. MAT 521: Advanced Linear Algebra
3. MAT 541: Numerical Analysis
4. MAT 513: Functional Analysis
5. MAT 523: Algebra (1)
6. MAT 531: Partial Differential Equations

II. Elective Courses:

List A

1. MAT 642: Advanced Numerical Analysis
2. MAT 645: Numerical Optimization
3. MAT 651: Graph Theory & Combinatorics
4. MAT 653: Combinatorial Optimization
5. MAT 661: Coding Theory & Cryptography
6. MAT 663: Mathematical and Computational Modeling
7. MAT 681: Selected Topics in Applied Mathematics (1)
8. MAT 683: Selected Topics in Applied Mathematics (2)

List B

1. MAT 624: Algebra (2)
2. MAT 626: Number Theory
3. MAT 628: Representation Theory
4. MAT 671: Topology
5. MAT 673: Algebraic Topology
6. MAT 675: Differential Geometry
7. MAT 685: Selected Topics in Pure Mathematics (1)
8. MAT 687: Selected Topics in Pure Mathematics (2)

III. Research Course:

1. MAT 699: Research Project

7. Program's Semester Wise Plan:

LEVEL 1	Course Code & Number	Course Name	Credit Hours	Lec.	Lab.	Tut.	Prerequisites
	MAT 511	Measure and Integration	4	3	0	1	
	MAT 521	Advanced Linear Algebra	4	3	0	1	
	MAT 541	Numerical Analysis	4	3	1	1	
			12	13			

LEVEL 2	Course Code & Number	Course Name	Credit Hours	Lec.	Lab.	Tut.	Prerequisites
	MAT 513	Functional Analysis	4	3	0	1	
	MAT 523	Algebra (1)	4	3	0	1	
	MAT 531	Partial Differential Equations	4	3	0	1	
			12	12			

LEVEL 3	Course Code & Number	Course Name	Credit Hours	Lec.	Lab.	Tut.	Prerequisites
	MAT xxx	Ellective Course (1)	4	x	x	x	
	MAT xxx	Ellective Course (2)	4	x	x	x	
	MAT xxx	Ellective Course (3)	4	x	x	x	
			12	x			

LEVEL 4	Course Code & Number	Course Name	Credit Hours	Lec.	Lab.	Tut.	Prerequisites
	MAT xxx	Ellective Course (4)	4	x	x	x	
	MAT xxx	Research Project	4	x	x	x	
			8	x			

8. Courses description

Core Courses

MAT 511 – Measure and Integration

Course Code & Number	Course Name	Credit Hours	Lec.	Lab.	Tut.	Prerequisites
MAT 511	**Measure and Integration**	**4**	**3**	**0**	**1**	

Syllabus:

Basics: Countable and uncountable sets, Axiom of Choice and Zorn's Lemma, Open and closed sets, Borel sets, σ-Algebras, Cauchy sequences and completeness of $\mathbb{R}$, continuity and uniform continuity.

Lebesgue Measure and Measurable Functions: Outer measure and measurable sets, σ-algebra of measurable sets, Countability of addidivity, Zero measure and Cantor's Set, Nonmeasurable sets, Measurable functions and their properties, Simple functions and the simple approximation theorem, Lusin's Theorem.

The Lebesgue's Integral: The Riemann's integral, The Lebesgue integral of a bounded measurable function, The Lebesgue integral of a measurable nonnegative function, The general Lebesgue integral and its properties, Convergence in measure, Fatou's lemma, Monotone Convergence and Beppo-Levi theorem, Lebesgue dominated Theorem, Radon-Nikodym Theorem, product of sigma-algebra and product measure, Fubini Theorem, Change of variable formula.

L^p Spaces: Definition and Properties, Cauchy-Schwartz, Minkowski's inequality and Holder inequalities, Riesz-Fischer theorem and completeness of L^p spaces.

References

1. H. L. Royden; *Real Analysis,* Prentice Hall, 4th ed. 2010. (Main Reference)
2. S. K. Berberian, *Measure and Integration*, AMS/Chelsea Publishing (2010).
3. 2. M. Capinski and P. E. Kopp, *Measure, Integral and Probability*, Springer.
4. M. R. Spiegel; *Real Variables: SCHAUM'S Series*; MacGraw Hill, 1st ed. 1990.
5. A. Kolmogorov and S. Fomin; *Introductory Real Analysis*; Dover Publications; 1st ed. 1975.

MAT 521 – Advanced Linear Algebra

Course Code & Number	Course Name	Credit Hours	Lec.	Lab.	Tut.	Prerequisites
MAT 521	**Advanced Linear Algebra**	**4**	**3**	**0**	**1**	

Syllabus:

Vector Spaces: review of Bases and Dimensions, Linear transformations and their matrices, Nullity and image, The solution space homogeneous linear system, The general solution of a non-homogeneous linear system, direct sum and quotient spaces, The rank nullity theorem and complements , Dual and double dual spaces.

Eigenvalues and Linear operators: Eigenvalues and eigenvectors, Characteristic polynomial and Cayley-Hamilton) theorem, Similarity and diagonalizations of Matrices, Invariant subspaces, Primary decompositions, Rational and Jordan canonical forms.

Inner Product Spaces: Inner products and norms, Orthonormality and (Gram-Schmidt) processes, The adjoint of a linear operator, Normal and self-adjoint Operators, Unitary and orthogonal operators and their Matrices, Orthogonal Projections and the Spectral Theorem.

Quadratic Forms: Bilinear and Quadratic Forms, Rank of a Quadratic Form, Equivalent Quadratic Forms, Diagonal Form of a Quadratic Form and Law of Inertia.

References

1. S. Friedberg, A. Insel and L. Spence; *Linear algebra*; Pearson, 4th Ed. 2002**. (Main Reference)**
2. K. Hoffman, R. Kunze; *Linear algebra*; Pearson, 2nd Ed. 1971.
3. J. Kwak and S. Hong; *Linear Algebra*; Birkhäuser Boston; 2nd Ed. 2004.

MAT 541 – Numerical Analysis

Course Code & Number	Course Name	Credit Hours	Lec.	Lab.	Tut.	Prerequisites
MAT 541	**Numerical Analysis**	**4**	**3**	**1**	**1**	

Syllabus:

Solving Equations: Newton's method, Brent method, Aitkin's Δ^2 method & Muller method; Error and convergence analysis.

Solving Linear Systems: Direct methods: Pivoting, LU; Error analysis; Iterative methods: Jacobi, Gauss-Seidel & SOR methods; Krylov subspaces methods (Conjugate gradient method, GMRES,...); Error and convergence analysis; Preconditioning.

Solving Ordinary Differential Equations: Implicit and Explicit Euler Methods; Local and global error; Taylor and Runge Kutta methods; Predictor Corrector methods; Implicit Methods and Stiff Equation; Multistep Method; Error and convergence analysis, stability, and consistency; Numerical methods for solving system of first order differential equations; Examples of implementation.

References:

1. J. Stoer and R. Burlish; *Introduction to numerical Analysis*; Springer-Verlag, 3rd Ed. 2010 **(Main Reference)**
2. T. Sauer; *Numerical Analysis*, Pearson 2012.
3. Richard L. Burden and J. Douglas Faires; *Numerical Analysis*, 8th Edition.

MAT 513 – Functional Analysis

Course Code & Number	Course Name	Credit Hours	Lec.	Lab.	Tut.	Prerequisites
MAT 513	**Functional Analysis**	**4**	**3**	**0**	**1**	

Syllabus:

Normed and Banach Spaces: Review of metric spaces, Completeness and Compactness, Normed spaces, Banach spaces, Embedding of a normed space in a Banach space, Equivalence of norms on finite dimensional normed spaces, Closeness and compactness in finite Banach spaces, Functional on normed spaces and the dual spaces, The normed space of linear operators. Sublinear and subadditive functional, The Hahn-Banach Theorem (Real and Complex) and its Corollaries, The separability theorem, Baire's category theorem , The uniform boundedness theorem, The open mapping theorem, Close graph theorem, The closed linear operator Theorem.

Hilbert spaces: Inner product and Hilbert spaces and their properties, Schwarz and triangle inequalities, Embedding of an inner Space in a Hilbert space, Subspaces of Hilbert spaces, Orthogonal complements and direct sums , The null space lemma , Orthonormal sets and sequences, Bessel inequality, Total sets and Total Orthonormal sets, Separable Hilbert spaces, Isomorphic property of the dimension, Functional on Hilbert spaces and Riesz's theorem, Hilbert's adjoint operator and its properties, Self-adjoint, unitary and normal operators.

Weak convergence and Sobolev spaces: weak convergence, weak derivatives, The Sobolev Space H^1, density of smooth functions, traces, Green's formula, H_0^1 space, imbedding theorems.

References:

1. E. Kreyszig; *Introductory Functional Analysis*; Wiley, 1st Ed. 1989. **(Main Reference)**
2. J. P. Aubin; *Applied Functional Analysis*, Wiley-Interscience, 2000.
3. J. Oden and, L. Demkowicz; Applied Functional Analysis; Chapman & Hall, 2nd Ed. 2009.
4. Y. Eidelman, V. Milman and A. Tsolomitis; *Functional Analysis: An Introduction*; American Mathematical Society; 1st Ed. 2004.

MAT 523 – Algebra (1)

Course Code & Number	Course Name	Credit Hours	Lec.	Lab.	Tut.	Prerequisites
MAT 523	**Algebra (1)**	**4**	**3**	**0**	**1**	

Syllabus:

Groups: Review of basics, Cauchy Theorem, Group Action, Orbits and stabilizers, The Class Equation, Sylow theorems, Direct Products and Direct Sums, Free abelian groups, Classifications of finitely generated abelian groups, Free Group, Presentation of groups, p-Grops, Nilpotent groups, Normal series and Jordan-Holders theorem, Solvable groups, Simple groups and simplicity of A_n .

Rings and Fields: Review of basics, Direct products and direct sums of rings, Polynomial Rings and their factorizations, Power Series Rings, Euclidean domains, PID and UFD, Gaussian rings, Field extensions, finite algebraic extensions, Geometric Constructions, Finite Fields.

Modules: Basics, Submodules and quotient modules, Direct products and direct sums of modules, Free modules and Finitely generated modules, Modules over PID, Chain conditions on Modules, Noetherian and Artinian Modules, composition series and Jordan-Holders theorem.

References

1. D. Dummit and R. Foote; *Abstract Algebra*; John Wiley, 3rd ed. 2003. **(Main Reference)**
2. D. Robinson; *An Introduction to Abstract Algebra*; De Gruyter, 2003.
3. T. W. Hungerford; *Algebra*; Springer, 1st ed. 1980.

MAT 531 – Partial Differential Equations

Course Code & Number	Course Name	Credit Hours	Lec.	Lab.	Tut.	Prerequisites
MAT 531	**Partial Differential Equations**	**4**	**3**	**0**	**1**	

Syllabus:

Generalities on PDEs: Classification of PDEs, Classical PDEs of mathematical physics**,** Definition of Initial-Boundary-Value Problems.

Analytic Solution of PDEs: Partial differential equation problems on finite domains: Separation of Variables and Fourier series; Partial differential equation problems on infinite domains: The Fourier Integral. The Fourier transforms. The Laplace transforms.

Elements of Distributions and Laplace operator: test functions and distributions, differentiation, multiplication by smooth functions; fundamental solution, weak solutions, Maximum principle; Harmonic functions. Green's Functions. Laplace's equation in polar and spherical coordinates.

References:

1. L. C. Evans; *Partial Differential Equations*; American Mathematical Society, 2nd Ed. 2010. **(Main Reference)**
2. R. Haberman; *Applied Partial Differential Equations with Fourier Series and Boundary Value Problems*; Pearson 2012.
3. R McOwen; *Partial Differential Equations: Methods and Applications*; 2nd Ed. 2002.

Elective Courses (List A)

MAT 642 – Advanced Numerical Analysis

Course Code & Number	Course Name	Credit Hours	Lec.	Lab.	Tut.	Prerequisites
MAT 642	**Advanced Numerical Analysis**	**4**	**3**	**1**	**1**	**MAT 513, MAT 541**

Syllabus:

Finite Differences: Approximation of first and second order derivatives, one-sided finite differences, analysis of truncation error, Higher order approximation, Example of $1D$ and $2D$ Poisson equation, Treatment of complex geometries, Evolution problems, analysis of stability.

Finite Elements: Galerkin approximation, mathematical Formulation of FEM, examples of elements: $P1$ elements; conforming and nonconforming elements, Convergence, Shape functions and stiffness matrix.

Computer Implementation: Applications to engineering problems.

Reference:

1. R. Leveque; Finite Difference Methods for Ordinary and Partial Differential Equations*; Steady-State and Time-Dependent Problems*, SIAM, 2007.
2. T. Chandrupatla and A. Belegundu; *Introduction to Finite Elements in Engineering*; International Edition, Pearson, 2012.
3. S. Brenner and R. Scott; *The Mathematical Theory of Finite Element Methods*; Springer, 3rd Ed. 2008.

MAT 645 – Numerical Optimization

Course Code & Number	Course Name	Credit Hours	Lec.	Lab.	Tut.	Prerequisites
MAT 645	**Numerical Optimization**	**4**	**3**	**1**	**1**	

Syllabus:

Convexity: convex sets, convex functions, characterization of convex functions, properties of convex functions.

Unconstrained problems: First order and second order local optimization conditions, the case convex programs.

Constrained problems: Optimal conditions for constrained problems, Karush Kuhn and Tucker theorem, Lagrange multipliers and duality, the case of convex problems.

Optimization algorithms and methods: The Simplex method, Deterministic direct search (Nelder Mead), descent-type methods (Gradient, Quasi-Newton methods, $BFGS$), genetic algorithms, evolution strategies; PSO), numerical implementation.

References:

1. J. Nocedal and S. J. Wright; *Numerical Optimization*; Springer, 2nd Ed. 2006. **(Main Reference)**
2. N. Gould and S. Leyffer; *An introduction to algorithms for nonlinear optimization*; Springer 2003.
3. S. Chandra, Jayadeva and Aparna Mehra; Numerical Optimization with Applications, Alpha Science Intl Ltd; 1st Ed., 2009.

MAT 651 – Graph Theory and Combinatorics

Course Code & Number	Course Name	Credit Hours	Lec.	Lab.	Tut.	Prerequisites
MAT 651	**Graph Theory and Combinatorics**	**4**	**3**	**0**	**1**	

Syllabus:

Graph Theory: Introduction to Graphs, Subgraphs, Connected Graphs, Trees, Nonseparable Graphs, Tree-Search Algorithms, Flows in Networks, Complexity of Algorithms, Connectivity, Planar Graphs, The Four-Color Problem, Matchings, Hamilton Cycles, Coverings and Packings, Electrical Networks, Integer Flows and Coverings, Stable Sets and Cliques, Colorings, Unsolved Problems.

Combinatorics: Counting principles, Arrangements and combinations, Numerations of object distributions, Generating functions and their coefficients, Partitions, Exponential generating functions and applications, Examples of recurrence relations, Homogeneous recurrence relations, nonhomogeneous recurrence relations, Solving recurrence relation by generating functions, Inclusion-Exclusion formula and applications, Arranging objects with restricted positions, Burside's theorem, The cycle index theorem, Polya's enumeration formula.

References:

1. J. A. Bondy and U. Murty; *Graph Theory*; Springer, 1st Ed. 2008. **(Main Reference)**
2. A. Tucker; *Applied Combinatorics*; Wiley and Sons, 6th Ed. 2012. **(Main Reference)**
3. R. J. Wilson; *Introduction to Graph Theory*; Pearson Education, Singapore, 4th Ed. 2003.
4. P. J. Cameron; *Combinatorics: Topics, Techniques, Algorithms*; Cambridge University Press, 1st Ed. 1995.

MAT 653 – Combinatorial Optimization

Course Code & Number	Course Name	Credit Hours	Lec.	Lab.	Tut.	Prerequisites
MAT 653	**Combinatorial Optimization**	**4**	**3**	**0**	**1**	

Syllabus:

Problems and Algorithms: Optimal Trees and Paths: Measuring Running Times, Minimum Spanning Trees, Shortest Paths**,** Maximum Flow Problems: Network Flows Problems, Maximum Flow Problems, Minimum Cut Problem, Multicommodity Flows, Minimum Cost Flow Problems.

Optimal Matchings: Matchings and Alternating Paths, Maximum Matchings, Minimum Weight Perfect Matchings, T-joins and Postman Problem.

Integral Polyhedra: Convex Hulls, Polytopes, Facets, Integral Polytopes, Total Unimodularity, Total Dual Integrality, Cutting Planes, Separation and Optimization.

The Traveling Salesman Problem: Introduction, Heuristics for the TSP, Lower Bounds, Cutting Planes, Branch and Bound.

Matroids: The Greedy Algorithm, Matroids: Properties, Axioms, Constructions, Matroids Intersection, Applications.

NP and NP-completeness: Introduction, Words, Problems, Algorithms and Running Time, The Class NP.

References:

1 W. Cook, W. Cunningham, W. Pulleyblank, and A. Schrijver; *Combinatorial Optimization*; Wiley-Blackwell, 1997. **(Main Reference)**

2 C. Papadimitriou and K. Steiglitz; *Combinatorial Optimization: Algorithms and Complexity*; Dover Publications Inc., 2000.

MAT 661 – Coding Theory & Cryptography

Course Code & Number	Course Name	Credit Hours	Lec.	Lab.	Tut.	Prerequisites
MAT 661	**Coding Theory & Cryptography**	**4**	**3**	**0**	**1**	**MAT 523**

Syllabus:

Basics and Linear Codes: Error detection, correction and decoding, Hamming distance and Distance of a code, MLD reliability, Linear Codes and their Basis, Generator matrix and parity-check matrix, Equivalence of linear codes, Encoding with a linear codes, Cosets of Linear Codes and the coset leader, Nearest neighbor decoding.

Bounds and Constructions of linear Codes: Optimal codes, extended codes and parity-check matrices, Bounds for codes and their types, Perfect Codes, Hamming Codes and their use, Golay Codes, Reed-Muller Codes and their use.

Cyclic Codes and Other Codes: Cuclic hamming codes, BCH Codes and their use, Codes over GF(2^n) , Reed-Solomon Codes, Quadratic-residue Codes, Hadamard matrix codes, Nordstrom,Robinson code, Preparata codes and Kerdock codes, Propagation rules of constructing Linear Codes, First order and higher Reed-Muller codes, Subfield Codes.

Classic Cryptography: Encryption Schemes, Symmetric key encryption, Fiestel Cipher and DES.

Public-Key Cryptography: (PKC): Algorithm and Complexity, Quadratic residues and quadratic reciprocity, Partiality testing, Discrete algorithm, Hash functions, RSA, Provable security and ELGamal, Cryptography Protocols (Diffe Hellman, Zero Knowledge and coin-tossing).

References:

1. D, Hankerson and others; *Coding Theory and Cryptography: The Essentials*; Marcel Dekker, 2nd Ed., 2000 **(Main Reference)**
2. S. Ling and C. Xing; *Coding Theory: A First Course*; Cambridge University Press, 1st ed. 2004.
3. J. van Lint; *Introduction to Coding Theory*; Springer 3rd Ed. 1998.
4. Shu Lin and D. Castello; *Error Correcting Codes*; Prentice Hal, 2nd ed. 2004.

MAT 663 – Mathematical and Computational Modeling

Course Code & Number	Course Name	Credit Hours	Lec.	Lab.	Tut.	Prerequisites
MAT 663	Mathematical and Computational Modeling	4	3	0	1	MAT 531

Syllabus:

Mathematical modeling tools: Needs and Techniques of mathematical modeling: Idea of mathematical modeling; Steps in mathematical modeling, Characteristics of mathematical modeling.

Case Studies: Models in mechanical vibration (Spring mass system, pendulum problems). Models in population dynamics (One species model, logistic model, growth model in time delays, Predator-Prey models, Volterra-Lotka models). Models of chemical processes, Electrical network and Diffusion processes, Trafic Flow Models.

Modeling dynamical systems: Differential equations and their numerical solutions, linear and nonlinear dynamics, stability, convergence, attractors.

Physical systems: System types and characteristics behaviour, Continuous-time, discrete Vs time and discrete, event systems, linear and nonlinear systems.

Exploration of behaviour through simulation: developing simulations of dynamical systems using Matlab /Simulation: representation and visualization of simulation experiments, analyzing behavioural characteristics for a range of classes of physical and computational systems *eg*. Predictor- prey models, evolutionary systems and cellular system.

References

1. J. N. Kapur; *Mathematical modeling*; Wiley eastern Ltd., 1994. **(Main Reference)**
2. M. M. Gibbons; *A concrete approach to Mathematical modeling*; John Wiley and Sons, 1995.
3. H. Neunzert and A. Siddiqui; *Topics in Industrial Mathematics*; Kluwer Academic Publishers, London, 2000.

MAT 681 – Selected Topics in Applied Mathematics I

Course Code & Number	Course Name	Credit Hours	Lec.	Lab.	Tut.	Prerequisites
MAT 681	Selected Topics in Applied Mathematics I	4	x	x	x	

MAT 683 – Selected Topics in Applied Mathematics II

Course Code & Number	Course Name	Credit Hours	Lec.	Lab.	Tut.	Prerequisites
MAT 683	Selected Topics in Applied Mathematics II	4	x	x	x	

Elective Courses (List B)

MAT 624 – Algebra (2)

Course Code & Number	Course Name	Credit Hours	Lec.	Lab.	Tut.	Prerequisites
MAT 624	**Algebra (2)**	**4**	**3**	**0**	**1**	**MAT 523**

Syllabus:

Rings: Power Series Ring, Localizations of commutative rings, Chain Conditions on Rings, Noetherian and Artinian Rings, Semi-simple Rings, Wedderburn-Artin theorem.

Commutative Rings and their Modules: Properties of commutative Noetherian rings, Primary Ideals and primary submodules, Noetherian Modules, Primary Decomposition, Reduced primary decomposition, Krull Intersection Theorem, Nakayama lemma, Hilbert Basis theorem, integral extension of commutative rings, Dedekind Domains.

Fields: Extension fields, Algebraic and transcendental elements, The isomorphism extension theorem, Splitting field extensions, Geometric constructions, Separable and normal field Extensions, The fundamental theorem of Galois, Cyclotomic extensions, Insolvability of the quintic.

References

1. T. W. Hungerford; *Algebra*; Springer, 1st Ed. 1980. **(Main Reference)**
2. D. Dummit and R. Foote; *Abstract Algebra*; John Wiley, 3rd Ed. 2003.
3. J. Rotman; *Advanced Modern Algebra*; American Mathematical Society; 2nd Ed. 2010.

MAT 626 – Number Theory

Course Code & Number	Course Name	Credit Hours	Lec.	Lab.	Tut.	Prerequisites
MAT 626	**Number Theory**	**4**	**3**	**0**	**1**	

Syllabus:

Linear and Quadratic Equations: Review of Congruencies Arithmetic, the linear congruence equation, The public-key coding system, Quadratic residues, Legendre symbol and its properties, Gauss lemma, Quadratic Reciprocity Law, Jacobi symbol, Worked examples and math software applications.

Quadratic forms: Definitions and basic properties, Equivalence of quadratic forms, Reduced quadratic forms , Quadratic representation, two squares sum representation, Sums of four squares sum representation and Lagrang's Theorem .

Continued Fractions: Definitions and basic properties,, Characterizing rationales by finite continued fractions, Finding a particular solution of the linear congruence equation, Continued fractions approximation of a real number, Purely periodic continued fraction, Pell's Equations.

References

1. A. Baker; *A Comprehensive Course in Number Theory*; Cambridge University Press, 2012. **(Main Reference)**

2. I. Niven, H. Zuckerman, H. Montgomery; *An Introduction to the Theory of Numbers*; Wiley 5th Ed., 1991.

2. C. Olds; *Continued Fractions*; Mathematical Association of America, 1992.

MAT 628 – Group Representation

Course Code & Number	Course Name	Credit Hours	Lec.	Lab.	Tut.	Prerequisites
MAT 628	**Group Representation**	**4**	**3**	**0**	**1**	**MAT 523**

Syllabus:

Group Representation: Definitions and examples, Equivalent Representations, Group Algebras, group algebras modules and submodules, Regular representation, Permutation Representation, Irreducible modules and completely reducible modules, Module homomorhisms and isomorphic modules, Mschke's theorem, Schur's Lemma and applications.

Group Characters: The Conjugacy class equation, Center of group algebra, Characters, Irreducible, regular and faithful Characters, Inner Products of Characters, The number of Irreducible Characters, Character tables, Row and column orthogonality relations of characters, Computing characters tables of small orders groups, Lifted characters, Finding linear characters by lifting, Tensor Product of representations and their characters, Characters of finite direct product of groups.

References

1. James & Liebeck; *Representations and Characters of Groups*; 2nd Ed., Cambridge, 2011. **(Main Reference)**
2. M. Burrow; *Representation Theory of Finite Groups*; Dover Publications 2011.
3. L. Dornhoff; *Group Representation theory - Part A*; Marcel Dekker 1971.

MAT 671 – Topology

Course Code & Number	Course Name	Credit Hours	Lec.	Lab.	Tut.	Prerequisites
MAT 671	**Topology**	**4**	**3**	**0**	**1**	**MAT 513**

Syllabus:

Basics: review of set theory, Definition and examples of toplogical spaces, equivalent topologies, Basis and Subbasis, Subspaces, Order topology, Closed sets and the closure of a set, Hausdorff spaces, Continuous functions, Homeomorphisms and topological properties, Product and box topologies.

Metric Spaces: Definition and important examples, Metrics on $\mathbb{R}^{\omega}$**,** The Induced metric topology, Metrizable tpological spaces, Sequences and the sequence lemma, Cauchy sequences and complete metric Spaces, Uniform convergence.

Connectednes & Compactness: Separation and connected topological spaces, Basic properties, products of connected spaces, Path connectedness, Connectedness in $\mathbb{R}$. Open covering and compact spaces, Basic properties, Hausdorff compact spaces, Compactness in $\mathbb{R}$**,** Lebesgue number lemma and the uniform convergence theorem, Compactness in $\mathbb{R}^{n}$, Tychonoff theorem, Limit and sequentially compactness, Local compactness and the one-point compactification.

Countability and Separation Axioms: Countability Axioms, Separation Axioms, Normal Spaces Urysohn Lemma and Urysohn Metrization Theorem.

References

1. J. R. Munkres; *Topology*; Pearson, 2nd Ed. 2000. **(Main Reference)**
2. B. Mendelson; *Introduction to Topology*; Dover Publications; 3rd Ed. 1990.
3. S. Willard; *General Topology*; Dover Publications 2004.

MAT 673 – Algebraic Topology

Course Code & Number	Course Name	Credit Hours	Lec.	Lab.	Tut.	Prerequisites
MAT 673	Algebraic Topology	4	3	0	1	MAT 671

Syllabus:

The Fundamental Group: Paths and Homotopy, Simply connected Spaces, The Fundamental Group $\pi_1(X)$ Quotient topology, Covering Spaces and Universal covering spaces, The Fundamental Group of the Circle, Retractions and Fixed Points, The Fundamental Theorem of Algebra, Borsuk-Ulam Theorem, Deformation retracts and Homotopy type, The induced homomorphism, The Fundamental Group of S^n , Fundamental groups of torus, double torus and Projective Plane. Higher Homotopy Groups $\pi_n(X)$, Homotopy Groups $\pi_n(X)$ of S^n.

Homology Groups: Geometric Complexes and Polyhedra; Orientation of Geometric Complexes; Chains, Cycles and boundaries; Simplicial Homology Groups $H_n(X)$; The Euler-Poincare Theorem; Homology Groups of S^n, Simplicial Approximation; induced homomorphisms, The Relation Between the group $H_n(X)$ and the group $\pi_n(X)$. Chain Derivation Lefschetz Fixed Point Theorem and general Brouwer Fixed Point Theorem; Relative Homology Groups; Singular Homology Theory; Axioms for Homology Theory.

References

1. J. Munkres; *Topology*; Pearson, 2nd ed. 2000. **(Main Reference)**
2. F. Croom; *Basic Concepts of Algebraic Topology*; Springer 1978. **(Main Reference)**
3. Allen Hatcher; *Algebraic Topology*; Cambridge University Press 2002.

MAT 675 – Differential Geometry

Course Code & Number	Course Name	Credit Hours	Lec.	Lab.	Tut.	Prerequisites
MAT 675	**Differential Geometry**	**4**	**3**	**0**	**1**	

Syllabus:

Plane curves: Curves' parameterizations, Arc length, Curvature and torsion, Frenet formulas, The local canonical form, Global properties of plane curves, Simple closed curves.

Regular Surfaces in $\mathbb{R}^3$: Surfaces' parameterizations, Regular surfaces and regular values, Change of parameters, A regular parametrized Surface, Tangent planes, Tangent spaces and normal vectors, The first fundamental form of a regular surface.

The Geometry of Gauss Map: Gauss map and its properties, Differential of the Gauss map, Meusnier's theorem, The second fundamental form of a regular surface, The maximum and the minimum normal curvatures, Gaussian and Mean curvatures, The Gauss map in local coordinates, Equations of Weingarten, Examples.

References

1. M. P. Do Carmo; *Differential Geometry of Curves and Surfaces*; Pearson, 1st Ed., 1976. **(Main Reference)**
2. Andrew Pressley; *Elementary Differential Geometry*; Springer-Verlag, 1st Ed., 2010.
3. Wolfgang Kühnel; *Differential Geometry: Curves, Surfaces, Manifolds*; American Mathematical Society; 2nd Ed. 2004.

MAT 685 – Selected Topics in Pure Mathematics (1)

Course Code & Number	Course Name	Credit Hours	Lec.	Lab.	Tut.	Prerequisites
MAT 685	Selected Topics in Pure Mathematics (1)	4	x	x	x	

MAT 687 – Selected Topics in Pure Mathematics (2)

Course Code & Number	Course Name	Credit Hours	Lec.	Lab.	Tut.	Prerequisites
MAT 687	Selected Topics in Pure Mathematics (2)	4	x	x	x	

Research Course

MAT 699 – Research Project

Course Code & Number	Course Name	Credit Hours	Lec.	Lab.	Tut.	Prerequisites
MAT 699	Research Project	4	-	-	-	Level 4 / Year 2