Title:

On Hamilton-Jacobi Equations

Abstract

Based mainly on the famous book of Evans [1], published on 2010, this research project aims to study Hamilton-Jacobi equations of the form:

$$\begin{cases} u_t + H(\nabla_x u) = 0, & \text{in } \mathbb{R}^n \times (0, \infty) \\ u(x, 0) = g(x), & \text{on } \mathbb{R}^n \times \{t = 0\}. \end{cases}$$

We raise and discuss some problematic questions related to the existence and uniqueness of "solution" and in what sense. For that, we involve there approaches of solutions in: the classical sense, the weak sense, and the viscosity sense. The dual connection between Hamiltonian and Lagrangian, allows us to obtain a variational solution involving Hopf-Lax formula and Legendre transform.

[1] Lawrence C.Evens:*Partial Differential Equations* . American Mathematical Society, RI, 2010.