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المملكة العربية السعودية
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كلية العلوم وعمادة البرامج التحضيرية

FINAL EXAM

COURSE: TRACK OF APPLIED SCIENCE (MAT 050)		
SEMESTER: SUMMER	**YEAR:** 1433/1434	**DURATION:** 120 min

	الشعبة	Answer Key	الاسم
	التوقيع	٠٥٠ رجض	الرقم الجامعي

INSTRUCTIONS

1) The exam contains **07 Pages** total (including the first pages!!) and **04 QUESTIONS.**
2) NO book, NO notes, NO Calculator.

	SCORE
QUESTION 1	______/10
QUESTION 2	______/12
QUESTION 3	______/08
QUESTION 4	______/10
TOTAL	______/40

لا لب

QUESTION 1: (10 Marks)

Let the functions $f(x) = \sqrt{3-3x}$ and $g(x) = \frac{1}{2x-3}$.

a) Find the domain of the functions f and g.

D_f

$3 - 3x \geq 0$

$3 \geq 3x$

$1 \geq x$

$D_f = (-\infty, 1]$

D_g

$2x - 3 = 0$

$2x = 3$

$x = \frac{3}{2}$

$D_g = \mathbb{R} - \{\frac{3}{2}\}$.

b) Find $(f+g)(-2)$ and compute the composition$(g \circ f)(x)$.

$(f+g)(-2) = f(-2) + g(-2)$

$= \sqrt{9} + \frac{1}{-4-3}$

$= 3 + \frac{1}{-7}$

$= 3 - \frac{1}{7}$

$= \frac{20}{7}$

$(g \circ f)(x) = g(f(x))$

$= g(\sqrt{3-3x})$

$= \frac{1}{2\sqrt{3-3x} - 3}$

c) Find the inverse of the function g.

Let $g(x) = y$

$\frac{1}{2x-3} = y$

$2xy - 3y = 1$

$2xy = 3y + 1$

$x = \frac{3y+1}{2y}$

$f^{-1}(x) = \frac{3x+1}{2x}$

3) Prove that $x + 3$ is a factor of the polynomial $2x^4 + 6x^3 + x^2 + 2x - 3$.

Let $f(x) = 2x^4 + 6x^3 + x^2 + 2x - 3$

$$f(-3) = 2(-3)^4 + 6(-3)^3 + (-3)^2 + 2(-3) - 3$$

$$= 162 - 162 + 9 - 6 - 3$$

$$= 0$$

$\therefore$ $(x+3)$ is a factor of $f(x)$

4) Find all possible rational zeros of the polynomial $4x^6 - 4x^2 + 3x - 2$.

$p = 1, 2$

$q = 1, 2, 4$

possible rational zeros $= \mp \frac{p}{q}$

$$= \left\{ \mp 1, \mp \frac{1}{2}, \mp \frac{1}{4}, \mp 2 \right\}$$

QUESTION 3: (8 Marks)

Solve the following equations:

a) $3^{2x-1} = \frac{1}{27}$

$3^{2x-1} = 3^{-3}$

$2x - 1 = -3$

$2x = -3 + 1$

$2x = -2$

$x = -1$

b) $(2x)^{-3}x^2 = \frac{1}{25}$

$\frac{1}{(2x)^3} \cdot x^2 = \frac{1}{25}$

$\frac{1}{8x^3} \cdot x^2 = \frac{1}{25}$

$\frac{1}{8x} = \frac{1}{25}$

$8x = 25$

$x = \frac{25}{8}$

c) $\log_x\left(\frac{1}{9}\right) = -2$

$x^{-2} = \frac{1}{9}$ by definition of logarithm

$\frac{1}{x^2} = \frac{1}{9}$

$x^2 = 9$

$x = \mp 3$

but $x = -3$ is refused $\Rightarrow \boxed{x = 3}$

d) $\log(x) = \log(4 - x)$

$x = 4 - x$ because logarithmic function is one to one

$2x = 4$

$x = 2$

2) Find the value of x using Cramer rule for the system

$$\begin{cases} 2x - y + z = 3 \\ 2x + y - 2z = 5 \\ x + y - z = 3 \end{cases}$$

$$\Delta = \left|\begin{matrix} 2 & -1 & 1 \\ 2 & 1 & -2 \\ 1 & 1 & -1 \end{matrix}\right| \begin{matrix} 2 & -1 \\ 2 & 1 \\ 1 & 1 \end{matrix} = (-2+2+2)-(2-4+1) = 3$$

$$\Delta x = \left|\begin{matrix} 3 & -1 & 1 \\ 5 & 1 & -2 \\ 3 & 1 & -1 \end{matrix}\right| \begin{matrix} 3 & -1 \\ 5 & 1 \\ 3 & 1 \end{matrix} = (-3+6+5)-(5-6+3) = 6$$

$$x = \frac{\Delta x}{\Delta} = \frac{6}{3} = 2.$$

3) Using Gauss-Jordan method, find the inverse of the matrix

$$A = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 3 & 2 \\ 0 & 2 & 1 \end{bmatrix}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ -1 & 3 & 2 & 0 & 1 & 0 \\ 0 & 2 & 1 & 0 & 0 & 1 \end{array}\right] \quad R_1 + R_2$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 3 & 2 & 1 & 1 & 0 \\ 0 & 2 & 1 & 0 & 0 & 1 \end{array}\right] \quad \frac{1}{3} R_2$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 2/3 & 1/3 & 1/3 & 0 \\ 0 & 2 & 1 & 0 & 0 & 1 \end{array}\right] \quad -2R_2 + R_3$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 2/3 & 1/3 & 1/3 & 0 \\ 0 & 0 & -1/3 & -2/3 & -2/3 & 1 \end{array}\right] \quad 2R_3 + R_2$$

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$\Longrightarrow$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1 & -1 & 2 \\ 0 & 0 & -\frac{1}{3} & -\frac{2}{3} & -\frac{2}{3} & 1 \end{array}\right] \quad -3R_3$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1 & -1 & 2 \\ 0 & 0 & 1 & 2 & 2 & -3 \end{array}\right]$$

$$\therefore A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & -1 & 2 \\ 2 & 2 & -3 \end{bmatrix}$$

check:

$$A \cdot A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 3 & 2 \\ 0 & 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ -1 & -1 & 2 \\ 2 & 2 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ -1-3+4 & 0-3+4 & 0+6-6 \\ 0-2+2 & 0-2+2 & 0+4-3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= I$$