بسم الله الرّحمن الرّحيم

Kingdom of Saudi Arabia
Ministry of Higher Education
Al-Imam Mohammed Ibn Saud
Islamic University
- College of Science -
Department: Maths & Stat.
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المملكة العربية السعودية
وزارة التّعليم العــــالي
جامعة الإمام محمّد بن سعود الإسلامية
-كلّيــــة العُــــلوم-
Course Name: Precalculus 2
Course : Math 060

Midterm 2

رياض ٦٠.

الشعبة	الرّقم الجامعي	إسم الطّالب
		Answer Key

يحتوي الاختبار على خمس صفحات

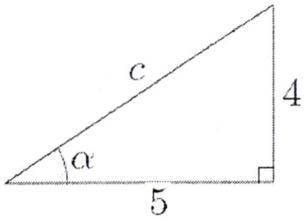
تعليمات هامّة

- يمنع استعمال الحاسبة.
- يمنع استعمال الجوّال.

د. سعود حامد الشريف

Question 1.

1. Consider the following right triangle.



Find the value of c and deduce the exact values of $\sin\alpha$, $\cos\alpha$ and $\tan\alpha$. **(4pts)**

Answer:

$c = \sqrt{25+16} = \sqrt{41}$

$\sin\alpha = \frac{4}{\sqrt{41}}$

$\cos\alpha = \frac{5}{\sqrt{41}}$

$\tan\alpha = \frac{4}{5}$

2. Let θ be in $[0, \pi]$ such that $\cos\theta = \frac{4}{5}$. Find the value of $\sin\theta$. **(2pts)**

Answer:

$\sin^2\theta + \cos^2\theta = 1$

$\sin^2\theta + \left(\frac{4}{5}\right)^2 = 1$

$\sin^2\theta = 1 - \frac{16}{25} = \frac{9}{25}$

$\therefore \sin\theta = \pm\sqrt{\frac{9}{25}} = \pm\frac{3}{5}$

$\therefore \sin\theta = +\sqrt{\frac{9}{25}} = +\frac{3}{5}$

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Question 2.

1. Evaluate each expression :

(a) $\cos(\frac{\pi}{3})\cos(\frac{\pi}{4}) - \sin(\frac{\pi}{3})\sin(\frac{\pi}{4})$. **(2pts)**

Answer:

$$\frac{1}{2}\cdot\frac{1}{\sqrt{2}} - \frac{\sqrt{3}}{2}\cdot\frac{1}{\sqrt{2}}$$

$$= \frac{1}{2\sqrt{2}} - \frac{\sqrt{3}}{2\sqrt{2}}$$

$$= \quad = \quad = \frac{1-\sqrt{3}}{2\sqrt{2}}$$

(b) $\sin(\frac{\pi}{3})\cos(\frac{\pi}{4}) - \cos(\frac{\pi}{3})\sin(\frac{\pi}{4})$. **(2pts)**

Answer:

$$\frac{\sqrt{3}}{2}\cdot\frac{1}{\sqrt{2}} - \frac{1}{2}\cdot\frac{1}{\sqrt{2}}$$

$$= \frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}} =$$

$$= \quad = \frac{\sqrt{3}-1}{2\sqrt{2}}$$

2. Deduce from (a) and (b) the exact values of $\cos\frac{7\pi}{12}$ and $\sin\frac{\pi}{12}$. **(2pts)**

Answer:

$$\cos\left(\frac{7\pi}{12}\right) = \cos\left(\frac{\pi}{3}+\frac{\pi}{4}\right) = \frac{1-\sqrt{3}}{2\sqrt{2}}$$

$$\sin\left(\frac{\pi}{12}\right) = \sin\left(\frac{\pi}{3}-\frac{\pi}{4}\right) = \frac{\sqrt{3}-1}{2\sqrt{2}}$$

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د. عبد الله الشمري

Question 3.

1. Solve in complex numbers the equation (2pts)

$$x^2 + 4x + 5 = 0.$$

Answer:

$$\frac{-b \pm \sqrt{\Delta}}{2 \cdot a} = \frac{-4 \pm \sqrt{-4}}{2 \cdot 1}$$

$$\Delta = \sqrt{b^2 - 4 \cdot a \cdot c}$$
$$= \sqrt{16 - 4 \cdot 1 \cdot 5}$$
$$= \sqrt{16 - 20}$$
$$= \sqrt{-4}$$

$$= \frac{-4 \pm \sqrt{4}\, i}{2}$$

$$= \frac{-4 \pm 2i}{2} = -2 \pm i$$

$$\therefore\ x^2 + 4x + 5 = 0$$
$$(x - (-2 + i))(x - (-2 - i)) = 0$$
$$(x + 2 - i) \cdot (x + 2 + i) = 0.$$

2. Write $i^{20}\sqrt{-9}$ in the form $a + bi$. (2pts)

Answer:

$$i^{20} \cdot \sqrt{-9} = i^{20} \cdot \sqrt{9}\, i = i^{20} \cdot 3 \cdot i^{1}$$
$$= i^{21} \cdot 3 = (i^4)^5 \cdot i \cdot 3$$
$$= (1)^5 \cdot 3 \cdot i$$
$$= 3i$$
$$= 0 + 3i$$

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د. سعود عايد الشمري

3. Write $\dfrac{1-2i}{3+i}$ in the form $a+bi$. **(2pts)**

Answer:

$$\frac{1-2i}{3+i} \times \frac{3-i}{3-i}$$

$$= \frac{3-i-6i+2i^2}{9+1}$$

$$= \frac{3-7i-2}{10} = \frac{1-7i}{10}$$

$$= \frac{1}{10} + \left(\frac{-7}{10}\right)i$$

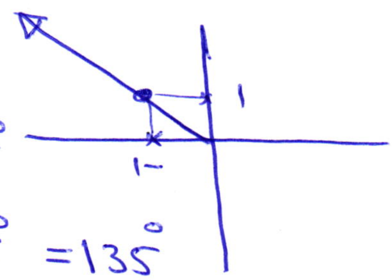
4. Write the complex number $z = -1+i$ in trigonometric form $r(\cos\theta + i\sin\theta)$. **(2pts)**

$$r = |-1+i| = \sqrt{(-1)^2+(1)^2} = \sqrt{1+1} = \sqrt{2}$$

$$a = r \cdot \cos\theta$$

$$-1 = \sqrt{2} \cdot \cos\theta$$

$$\frac{-1}{\sqrt{2}} = \cos\theta \Rightarrow \theta = \frac{\pi}{4} = 45^\circ$$



But θ in qadrant II $\Rightarrow 180 - 45^\circ = 135^\circ$

$$\therefore r(\cos\theta + i\sin\theta) = \sqrt{2}(\cos 135^\circ + i\sin 135^\circ).$$

مع دعواتنا للجميع بالتوفيق -